This worksheet examines the properties and graphs of reciprocal functions, emphasising how the graph of a hyperbola, such as $\frac{1}{x}$, displays features (asymptotes, symmetry, and branch behaviour) which distinguish it from many other function types.

Easy Questions

1. Find the vertical and horizontal asymptotes of $\frac{1}{x}$.
2. Calculate the corresponding y-values for $y = \frac{1}{x}$ when $x = 1$, $x = 2$, $x = -1$ and $x = -2$.
3. Determine the domain and range of $y = \frac{1}{x}$.
4. Show that $y = \frac{1}{x}$ is an odd function by verifying that $f(-x) = -f(x)$.
5. Solve the equation $\frac{1}{x} = 2$ for x.

Intermediate Questions

6. Find the vertical and horizontal asymptotes of $y = \frac{3}{x}$.
7. Determine the domain and range of $y = \frac{-2}{x}$.
8. For $y = \frac{4}{x}$, calculate the y-values when $x = 2$, 4, -2 and -4.
9. Sketch the graph of $y = \frac{1}{x}$ and identify the two separate branches. Label the vertical and horizontal asymptotes.
10. Explain why every point on the curve $y = \frac{1}{x}$ satisfies $x\,y = 1$.
11. Determine the constant k such that the reciprocal function $y = \frac{k}{x}$ passes through the point $(2, 5)$.

12. Discuss the symmetry of $y = \dfrac{k}{x}$ and explain why its graph is symmetric with respect to the origin.

13. For the function $y = \dfrac{2}{x}$, find $\lim\limits_{x \to 0^+} y$ and $\lim\limits_{x \to 0^-} y$, and justify your answers.

14. Solve the equation $\dfrac{1}{x} = -4$ for x.

15. Solve the equation $\dfrac{2}{x} = 10$ for x.

16. A hyperbolic relationship is modelled by $y = \dfrac{C}{x}$. If $y = 6$ when $x = 2$, find the value of C, and then determine y when $x = -5$.

17. Given $y = \dfrac{8}{x}$, find the value of x for which $y = 2$.

18. For the function $y = \dfrac{-3}{x}$, describe the change in y as x increases from -5 to -1.

19. For $y = \dfrac{5}{x}$, first determine the value of x for which $y = 10$, then find y when x is half that value.

20. For $y = \dfrac{-4}{x}$, show that the product $x\,y$ is constant for any point on the curve and state its value.

Hard Questions

21. Prove algebraically that $y = \dfrac{1}{x}$ is an odd function by showing that $f(-x) = -f(x)$.

22. Let $f(x) = \dfrac{k}{x}$ be a reciprocal function. Given that $f(3) = 7$, determine the value of k and then compute $f(-3)$.

23. Show that every point (x, y) on the graph of $y = \dfrac{k}{x}$ satisfies $x\,y = k$. Explain the significance of this relation in describing the graph of a hyperbola.

24. Determine the point(s) of intersection between $y = \dfrac{6}{x}$ and $y = x - 2$. Show all algebraic steps.

25. For the function $y = \dfrac{3}{x}$, compute $\lim\limits_{x \to \infty} y$ and $\lim\limits_{x \to -\infty} y$. Provide a justification for your answers.

26. Consider the function $f(x) = \dfrac{k}{x}$ with vertical asymptote at $x = 0$ and horizontal asymptote at $y = 0$. Explain why the graph never intersects either axis.

27. A rectangular hyperbola is defined by the equation $x\,y = k$. Derive the equations of its asymptotes and discuss their influence on the shape of the graph.

28. For $f(x) = \dfrac{5}{x}$, prove that the mid-point of the line segment joining a point $(x, \frac{5}{x})$ and its reflection about the origin is always $(0, 0)$.

29. Given $f(x) = \dfrac{2}{x}$, find the value of x for which $f(x)$ equals the arithmetic mean of $f(1)$ and $f(3)$.

30. A gas obeys the relationship $P \cdot V = k$, where P represents pressure and V represents volume. If $P = 2$ when $V = 5$, determine k and then find P when $V = 8$. Discuss briefly how this relationship illustrates the hyperbolic nature of the graph.