ILLAWARRA TUTORING

In this worksheet you will investigate how reflecting a function over an axis changes its graph, enhancing your understanding of symmetry.

Easy Questions

1. Write in a sentence what it means to reflect a function over the x-axis. Then, using the function $f(x) = x^2$, write the equation of the graph after reflecting it over the x-axis.

2. Explain what it means to reflect a function over the y-axis. Then, given $f(x) = 2x + 1$, write the equation of the reflected function.

3. State the general transformation rule for reflecting any function $f(x)$ over the x-axis and over the y-axis.

4. Given $f(x) = 3$, reflect the function over the x-axis and write the resulting function.

5. Identify which axis of reflection, if any, leaves the graph of $f(x) = |x|$ unchanged. Explain your answer in one or two sentences.

Intermediate Questions

6. Consider $f(x) = x^2$. Describe in one or two sentences what the graph looks like when reflected over the x-axis and when reflected over the y-axis. Indicate if the reflected graph is the same as the original in either case.

7. Given $f(x) = \sqrt{x}$, reflect the function over the y-axis. Write the equation of the reflected function and state its domain.

8. Reflect the function $f(x) = x - 4$ over the x-axis. Write the new function rule.

9. For $f(x) = \dfrac{1}{1 + x^2}$, find the equation of the function after reflecting it over the x-axis.

10. Find the equation for the reflection over the y-axis of $f(x) = 2x + 3$.

11. Reflect the function $f(x) = |x|$ over the x-axis and write down the resulting function. Does the graph change? Explain briefly.

12. Given $f(x) = x^3$, reflect it over the y-axis. Write the equation of the reflected function and state its domain.

13. Describe what happens to the coordinates of the point $(3, -2)$ when it is reflected over the x-axis.

14. Reflect the function $f(x) = \sqrt{x}$ over the x-axis and write the equation of the new function.

15. For the function $f(x) = \dfrac{1}{x-2}$, reflect it over the y-axis. Write the equation of the reflected function and state any restrictions on its domain.

16. The graph of $f(x) = -x^2$ is given. Describe in a few sentences what the graph looks like when reflected over the y-axis.

17. Reflect the function $f(x) = 3-x$ over the x-axis and write your answer as a function of x.

18. For the function $f(x) = -|x|$, reflect it over the y-axis. Write the equation of the image and comment on its appearance relative to the original graph.

19. Find the function that results when $f(x) = \dfrac{2}{x+1}$ is reflected over the x-axis.

Hard Questions

20. Derive a general rule for reflecting any function $f(x)$ over the x-axis and over the y-axis. Write a short algebraic justification for each rule.

21. Let $f(x) = \dfrac{1}{x^2}$ where $x \neq 0$. Reflect $f(x)$ over the x-axis and write the equation of the new function. Discuss any changes in the function's symmetry.

22. Given $f(x) = x + 5$, determine the equation of its reflection over the y-axis. Then, on a separate sheet of paper, sketch both the original and the reflected graphs, labelling key points.

23. Prove algebraically that the function $f(x) = x^2$ remains unchanged when reflected about the y-axis. Use appropriate substitution and simplification in your proof.

24. For the function $f(x) = \sqrt{x+2}$, reflect it over the y-axis. Write the equation for the reflected function and carefully determine its domain.

25. Let $g(x) = |x-1|+3$. Reflect $g(x)$ over the x-axis and write the resulting function. Explain any changes to the vertical displacement.

26. Given $f(x) = \dfrac{1}{x-3}$, reflect it over the x-axis to obtain a new function. Identify the location of the vertical asymptote of the reflected function and explain your reasoning.

27. Prove that for any point (a, b), its reflection over the x-axis is $(a, -b)$ and its reflection over the y-axis is $(-a, b)$. Provide a short justification using coordinate geometry.

28. Let $f(x)$ be an increasing function. Show that the function $g(x)$ defined as the reflection of $f(x)$ over the x-axis is a decreasing function. Provide an algebraic explanation to support your answer.

29. Consider $f(x) = \dfrac{2x}{x+1}$. Reflect it over the x-axis to obtain a new function. Write the expression for this reflected function and discuss any restrictions on the domain.