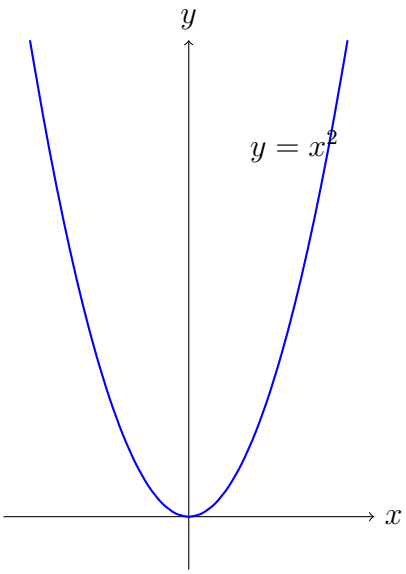
In this worksheet you will investigate how reflecting a function over an axis changes its graph. You will explore reflections over the vertical and horizontal axes and consider the effect on the equations, graphs and domains of functions.

Easy Questions

1. Write the equation obtained when reflecting the function $f(x) = 5$ over the x-axis.
2. Given $f(x) = x + 1$, find the equation for its reflection over the y-axis.
3. The graph of $f(x) = x^2$ is shown below. Sketch the graph of its reflection over the x-axis.



4. Write the equation for the reflection over the y-axis of $f(x) = \sqrt{x}$. Discuss any change in its domain.
5. Explain why the graph of $f(x) = |x|$ remains unchanged when reflected over the y-axis.

Intermediate Questions

6. If a point (a, b) lies on the graph of a function f, state the coordinates of its reflection over the x-axis.

7. If a point (a, b) lies on the graph of a function f, state the coordinates of its reflection over the y-axis.

8. Given $f(x) = 2x - 3$, find the equation of the function obtained by reflecting over the x-axis.

9. For the function $f(x) = 2x - 3$, determine the equation for its reflection over the y-axis.

10. For $f(x) = x^2$, explain why its reflection over the y-axis results in the same function.

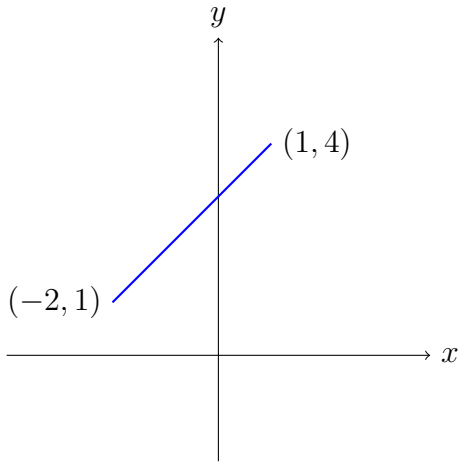
11. For $f(x) = x^3$, find the equation of its reflection over the y-axis.

12. Given $f(x) = \dfrac{1}{x+2}$, write the equation of the function after reflecting it over the y-axis.

13. Find the equation of the function obtained by reflecting $f(x) = \sqrt{x+1}$ over the x-axis.

14. Let $f(x) = |x|$. Reflect the function first over the x-axis and then over the y-axis. Write the final equation.

15. The diagram below shows a function f consisting of a line segment joining the points $(-2, 1)$ and $(1, 4)$. Using the diagram as a guide, sketch the graph of the reflection of f over the x-axis.



16. Prove that reflecting an even function f over the y-axis produces the same graph as f itself.

17. Given $f(x) = 2^x$, write the equation for its reflection over the y-axis.

18. For $f(x) = \ln(x)$ with domain $(0, \infty)$, determine the equation and domain of its reflection over the y-axis.

19. Given $f(x) = x^2 - 4$, find the equation for its reflection over the x-axis.

20. If the point $(3, 5)$ lies on the graph of f, list the coordinates of its reflections over the x-axis, the y-axis, and over both axes.

Hard Questions

21. Let $f(x) = x^2 - 6x + 5$. Find the equation for the reflection of f over the x-axis. Then complete the square to express your answer in vertex form.

22. Given $f(x) = \dfrac{x-2}{x+3}$, find the equation of the function obtained by reflecting over the y-axis. Simplify your expression.

23. For $f(x) = \sqrt{4-x}$, find the equation of its reflection over the y-axis and state the resulting domain.

24. Let $f(x) = \dfrac{1}{x^2-1}$. Reflect f over the x-axis to obtain a new function. Describe how the vertical asymptotes change, if at all.

25. Prove that reflecting an odd function over both the x-axis and the y-axis yields the original function.

26. Given $f(x) = e^x$, consider the function $g(x) = f(-x)$. Show that g is the reflection of f over the y-axis and discuss its asymptotic behaviour.

27. Compute the reflection over the x-axis of $f(x) = \sin(x)$ for $x \in [0, 2\pi]$. Describe briefly the symmetry properties of the sine function.

28. For $f(x) = \cos(x)$, find the equation of its reflection over the y-axis and prove that the resulting function is identical to f.

29. Suppose $f(x) = \dfrac{x^2+3x+2}{x+1}$. Simplify f and then find the equation for its reflection over the x-axis in simplest form.

30. Consider an arbitrary function $f(x)$. Prove that reflecting f over the x-axis and then over the y-axis is equivalent to rotating its graph 180° about the origin. Provide a detailed justification.