ILLAWARRA TUTORING

In this worksheet you will apply multiple transformations simultaneously to a given function and determine how these affect its graph. Remember to work carefully through each transformation step by step.

Easy Questions

1. The function is given by $f(x) = x^2$. Write the expression for the function obtained after reflecting $f(x)$ in the x-axis and then shifting it vertically up by 3 units.

2. The function is $f(x) = x$. Write the expression for the new function formed by shifting $f(x)$ horizontally left by 4 units and then stretching it vertically by a factor of 2.

3. The function is $f(x) = x^2$. Write an expression for the function obtained when $f(x)$ is shifted right by 3 units and shifted vertically down by 2 units.

4. The function is $f(x) = x$. Write the expression for the transformed function when you reflect $f(x)$ about the y-axis and then shift it vertically up by 2 units.

5. The function is $f(x) = \sqrt{x}$. Write an expression for the function obtained after shifting $f(x)$ horizontally left by 1 unit and vertically up by 4 units.

Intermediate Questions

6. The function is $f(x) = x^2$. Write the expression for the function obtained by first applying a horizontal stretch by a factor of 2, then a vertical stretch by a factor of 3, and finally shifting it vertically up by 1 unit.

7. The function is $f(x) = \sqrt{x}$. Write the expression for the function after applying a horizontal compression by a factor of 3 and reflecting it in the x-axis.

8. The function is $f(x) = x^3$. Write the expression for the function obtained by reflecting it over the y-axis, then shifting it horizontally right by 2 units and vertically down by 1 unit.

9. The function is $f(x) = |x|$. Write the expression for the function when it is stretched vertically by a factor of 4 and shifted horizontally left by 2 units.

10. The function is $f(x) = x^2$. Write an expression for the function obtained by horizontally compressing $f(x)$ by a factor of $\frac{1}{2}$ and then shifting it horizontally right by 3 units.

11. The function is $f(x) = \sqrt{x}$. Write the expression for the new function after reflecting it in the x-axis, shifting it horizontally right by 4 units, and then shifting it vertically down by 3 units.

12. The function is $f(x) = x^2$. Write the expression for the function obtained by stretching it vertically by a factor of 2, shifting it horizontally left by 1 unit, and then reflecting it in the x-axis.

13. The function is $f(x) = \sqrt{x}$. Write an expression for the function when it is horizontally stretched by a factor of 2 and shifted vertically up by 5 units.

14. The function is $f(x) = x^3$. Write the expression for the function obtained by reflecting it about the y-axis and then compressing it horizontally by a factor of $\frac{1}{2}$.

15. The function is $f(x) = |x|$. Write the expression for the function after stretching it vertically by factor 1.5, shifting it horizontally right by 2 units, and reflecting it in the x-axis.

16. The function is $f(x) = x^2$. Write an expression for the function obtained by horizontally stretching it by a factor of 3, shrinking it vertically by a factor of $\frac{1}{2}$, and then shifting it vertically down by 4 units.

17. The function is $f(x) = \sqrt{x}$. Write the expression for the function after shifting it horizontally left by 3 units and then reflecting it about the y-axis.

18. The function is $f(x) = x^3$. Write an expression for the function obtained by stretching it vertically by a factor of 3, shifting it horizontally left by 1 unit, and then reflecting it in the y-axis.

19. The function is $f(x) = |x|$. Write the expression for the function when it is shifted horizontally right by 3 units, stretched vertically by a factor of 2, and then shifted vertically down by 1 unit.

20. The function is $f(x) = x^2$. Write an expression for the function obtained by reflecting it in the x-axis, compressing it horizontally by a factor of $\frac{1}{2}$, and shifting it vertically up by 6 units.

Hard Questions

21. The function is $f(x) = (x+1)^2$. Apply the following transformations in order: shift $f(x)$ 3 units to the left, reflect it in the x-axis, stretch it vertically by a factor of 4, and finally compress it horizontally by a factor of $\frac{1}{2}$. Write the resulting function.

22. The function is $f(x) = \sqrt{x-2}$. Apply the following transformations in order: reflect $f(x)$ in the x-axis, shift it horizontally right by 4 units, shift it vertically down by 5 units, and then perform a horizontal stretch by a factor of 2. Write the expression for the transformed function.

23. The function is $f(x) = x^3$. Transform $f(x)$ by reflecting it over the y-axis, shifting it horizontally left by 2 units, stretching it vertically by a factor of 5, and finally shifting it vertically up by 3 units. Write the resulting expression.

24. The function is $f(x) = |x - 1|$. Apply the following transformations in order: compress it horizontally by a factor of $\frac{1}{2}$, shift it horizontally left by 3 units, reflect it in the x-axis, and then shift it vertically up by 4 units. Write the final expression.

25. The function is $f(x) = x^2$. Write the expression for the function obtained by applying a horizontal stretch by factor 3, a vertical shrink by factor $\frac{1}{4}$, reflecting it in the x-axis, shifting it horizontally left by 2 units, and finally shifting it vertically down by 7 units.

26. The function is $f(x) = \sqrt{x}$. Find the expression for the function obtained by shifting $f(x)$ horizontally right by 5 units, then compressing it horizontally by a factor of $\frac{1}{2}$ (note the order), stretching it vertically by a factor of 3, and finally shifting it vertically down by 1 unit.

27. The function is $f(x) = x^3$. Apply the following transformations in order: reflect the function in the y-axis, reflect it in the x-axis, shift it horizontally right by 1 unit, shift it vertically up by 2 units, and finally stretch it horizontally by a factor of 2. Write the resulting function.

28. The function is $f(x) = |x|$. Write the expression for the function obtained by applying a vertical stretch by a factor of 2, reflecting it in the horizontal axis (i.e. in the x-axis), shifting it horizontally left by 4 units, and then shifting it vertically down by 3 units.

29. The function is $f(x) = x^2$. Determine the expression for the function that results from shifting $f(x)$ horizontally right by 2 units, stretching it vertically by a factor of 3, compressing it horizontally by a factor of $\frac{1}{2}$, reflecting it in the x-axis, and finally shifting it vertically down by 5 units.

30. The function is $f(x) = \sqrt{x + 3}$. Apply the following transformations in order: stretch $f(x)$ horizontally by a factor of 3, shift it horizontally left by 2 units, reflect it in the x-axis, shift it vertically up by 6 units, and shrink it vertically by a factor of $\frac{1}{2}$. Write the final expression.