ILLAWARRA TUTORING

In this worksheet you will explore combined transformations and see how applying multiple changes at once affects a function's graph. You will practice writing function expressions arising from several transformations as well as explaining the order and effects of these transformations.

Easy Questions

1. The function $f(x) = x^2$ is given. Apply an upward translation by 2 units. Write the expression for the transformed function and sketch its graph on pen and paper.

2. The function $f(x) = x^2$ is given. Write the expression for $g(x)$ if you reflect $f(x)$ in the x-axis and then translate it upward by 3 units.

3. Given $f(x) = x$, apply a vertical stretch by a factor of 2 followed by a downward translation by 1 unit. Write the expression for the resulting function $g(x)$.

4. For $f(x) = x^2$, the function $g(x) = (x-2)^2 + 4$ is obtained by applying combined transformations. In plain text, describe the transformations applied.

5. Given $f(x) = x + 2$, reflect it across the y-axis and then translate it 3 units to the left. Write the expression for the resulting function.

Intermediate Questions

6. The function $f(x) = x^2$ is given. Apply a horizontal translation 4 units to the left and a vertical stretch by a factor of 3. Write the expression for $g(x)$.

7. Given $f(x) = x^2$, apply a vertical translation downward by 5 units and a horizontal translation 2 units to the right. Write the expression for $g(x)$.

8. Consider $f(x) = x$. Perform a horizontal stretch by a factor of 2 (i.e. replace x with $\frac{x}{2}$) and then translate the graph upward by 3 units. Write the resulting function.

9. Given $f(x) = x^2$, first reflect it in the x-axis and then apply a horizontal compression by a factor of $\frac{1}{2}$. Write the expression for $g(x)$.

10. For $f(x) = x^2$, apply a horizontal translation 3 units to the right, then reflect the function in the y-axis, and finally translate it downward by 2 units. Write the expression for $g(x)$.

11. Given $f(x) = x + 1$, reflect it in the x-axis and then apply a vertical stretch by a factor of 4. Write the resulting function.

12. The function $f(x) = x^2$ is provided. Apply a vertical compression by a factor of $\frac{1}{2}$ and then translate it horizontally 1 unit to the left. Write the expression for $g(x)$.

13. Let $f(x) = |x|$. Apply a horizontal translation 2 units to the right and a vertical translation 1 unit downward. Write the expression for the transformed function $g(x)$.

14. Consider $f(x) = x^2$. Apply the following transformations in order: reflect the graph in the x-axis, apply a horizontal stretch by a factor of 3, and translate the graph upward by 4 units. Write the expression for $g(x)$.

15. Given $f(x) = x$, apply a horizontal translation 5 units to the left, then a vertical stretch by a factor of 2, and finally reflect the graph in the x-axis. Write the expression for $g(x)$.

16. Sketch, on pen and paper, the graph of $g(x) = -2(x - 1)^2 + 3$ obtained from $f(x) = x^2$ by applying combined transformations. Clearly label the vertex.

17. The function $f(x) = x^2$ is transformed to $g(x) = 3 - (x + 2)^2$. List the sequence of transformations (in order) that maps $f(x)$ to $g(x)$.

18. For $f(x) = x^2$, apply the following sequence: a horizontal compression by a factor of 2, a reflection in the x-axis, a vertical compression by a factor of $\frac{1}{4}$, and a vertical translation downward by 1 unit. Write the resulting function $g(x)$.

19. Given $f(x) = x^2$, the function $g(x) = 2(x - 3)^2 - 7$ is obtained by combined transformations. State the order of the transformations applied.

20. Starting from $f(x) = x^2$, explain the order of transformations used to obtain $g(x) = -3(x - 2)^2 + 5$. Write a brief explanation emphasising each step.

Hard Questions

21. Given $f(x) = x^3$, apply the following transformations in order: reflect in the y-axis, apply a horizontal stretch by a factor of 2, a vertical compression by a factor of $\frac{1}{2}$, and then shift the graph upward by 4 units. Write the expression for $g(x)$ in simplified form.

22. Let $f(x) = x^2$. Find the function $g(x)$ that results from applying these transformations (in any order): a horizontal translation 4 units to the left, a vertical reflection, a vertical stretch by a factor of 5, and a vertical translation downward by 6 units.

23. Starting with $f(x) = x$, show step-by-step how to obtain $g(x) = -\frac{(x - 9)}{3} + 2$. Clearly indicate the transformations at each stage and explain the composite process.

24. Given $f(x) = |x|$, the function $g(x) = -\frac{1}{2}|x+4|+3$ is obtained. List all the transformations applied (including reflections, stretches/compressions, and translations) along with the order in which they are applied.

25. The quadratic $f(x) = x^2$ is transformed into $g(x) = 4 - 2(x+1)^2$. Identify the sequence of transformations and explain how the vertex of $f(x)$ is affected by these transformations.

26. Starting with $f(x) = x^2$, derive the transformation that produces $g(x) = (x-3)^2+2$. Then, for the point $(5, 6)$ on $g(x)$, determine its corresponding pre-image on $f(x)$ (i.e. the original point before transformation).

27. Consider $f(x) = x^2$. Suppose $g(x)$ is obtained by first reflecting $f(x)$ in the x-axis, then applying a vertical stretch by a factor of 2, followed by a horizontal compression by a factor of $\frac{1}{2}$, and finally translating the graph upward by 3 units. Write $g(x)$ in simplified form.

28. Draw a diagram that illustrates the effect of applying a horizontal shift 3 units to the left and a vertical translation 2 units upward on the graph of $f(x) = x^2$.

29. Suppose $f(x) = x^2$ is transformed into $g(x) = k(x-h)^2 + m$ through combined transformations. If the vertex of $g(x)$ is $(2, -1)$ and the graph passes through $(4, 7)$, determine the values of k, h, and m.

30. Compare $g(x) = -2(x-1)^2 + 3$ with the function obtained by taking $h(x) = -2x^2 + 3$ and then shifting it 1 unit to the right, namely $h(x-1)$. Discuss in detail whether these two functions are equivalent and explain how the order of applying transformations can affect the resulting graph.