ILLAWARRA TUTORING

This worksheet focuses on combined transformations. You will apply several transformations such as translations, reflections, stretches and compressions to a functions graph and describe in both algebraic and graphical ways how these transformations affect the graph.

Easy Questions

1. For the base function
$$f(x) = x^2,$$
state the transformation that produces
$$g(x) = 2x^2.$$

2. For the base function
$$f(x) = x^2,$$
describe the transformation applied to obtain
$$g(x) = (x-3)^2.$$

3. For
$$f(x) = x^2,$$
list the transformations needed to get
$$g(x) = (x+2)^2 + 5.$$

4. For
$$f(x) = x^2,$$
describe the transformation which produces
$$g(x) = -x^2.$$

5. Write down in order the transformations needed to change
$$f(x) = x^2$$
into
$$g(x) = 3(x-1)^2 - 4.$$

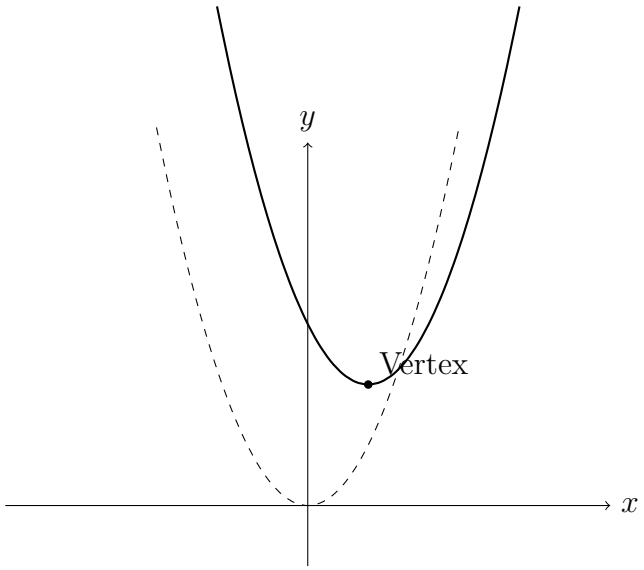
Intermediate Questions

6. For
$$f(x) = x^2,$$
describe all the transformations applied to obtain
$$g(x) = -2(x+1)^2 + 3.$$

7. Explain the order of operations when applying a horizontal translation, a vertical reflection, and a vertical stretch to a base function.

8. Given
$$f(x) = x^2,$$
sketch the graph of
$$g(x) = -\frac{1}{2}(x+3)^2 + 4.$$
On your graph, clearly indicate the effects of the reflection, stretch and translation.

9. Below is a diagram in TikZ showing a transformation of
$$f(x) = x^2.$$
Explain which transformations were applied.



10. Starting with
$$f(x) = x^2,$$
derive an expression for a function
$$g(x)$$

that is produced by applying the following transformations in order: a horizontal reflection, a horizontal stretch by a factor of

$$2,$$

a vertical stretch by a factor of

$$3,$$

and an upward translation of

$$4.$$

Show all your working.

11. For

$$g(x) = 2(x+4)^2 - 6$$

describe each transformation that has been applied to

$$f(x) = x^2.$$

12. Given

$$f(x) = \sqrt{x},$$

describe in order the effect of the transformations that produce

$$g(x) = \sqrt{3(x-2)} + 1.$$

13. For the base function

$$f(x) = |x|,$$

list the sequence of transformations that produces

$$g(x) = -|2x - 4| + 3.$$

14. Starting with

$$f(x) = x^2,$$

show algebraically the transformation sequence to obtain a graph that is reflected over the x-axis and then translated left by

$$3$$

units and down by

$$2$$

units.

15. Given
$$f(x) = x,$$
if you reflect it about the y-axis and apply a vertical stretch by a factor of
$$4,$$
what is the resulting function?

16. If
$$f(x) = x,$$
find the expression for
$$g(x) = -2f(1 - x) + 5.$$
Then, explain each transformation step.

17. Derive the function
$$h(x)$$
that results from applying a horizontal translation left
$$2$$
units, followed by a vertical scaling by
$$\frac{1}{3}$$
and a vertical translation of
$$-7$$
to the base function
$$f(x) = x^2.$$
Write
$$h(x)$$
in simplified form.

18. For
$$f(x) = x^2,$$
determine the transformations that yield
$$g(x) = \frac{1}{4}(x - 5)^2.$$

19. Explain how the order of the transformations affects the final graph using the example of
$$f(x) = x^2$$
and
$$g(x) = [2(x - 3)]^2 - 8.$$

20. With
$$g(x) = -3\left[\frac{x+1}{2}\right]^2 + 4,$$
demonstrate the step by step series of transformations that would take
$$f(x) = x^2$$
to
$$g(x).$$

Hard Questions

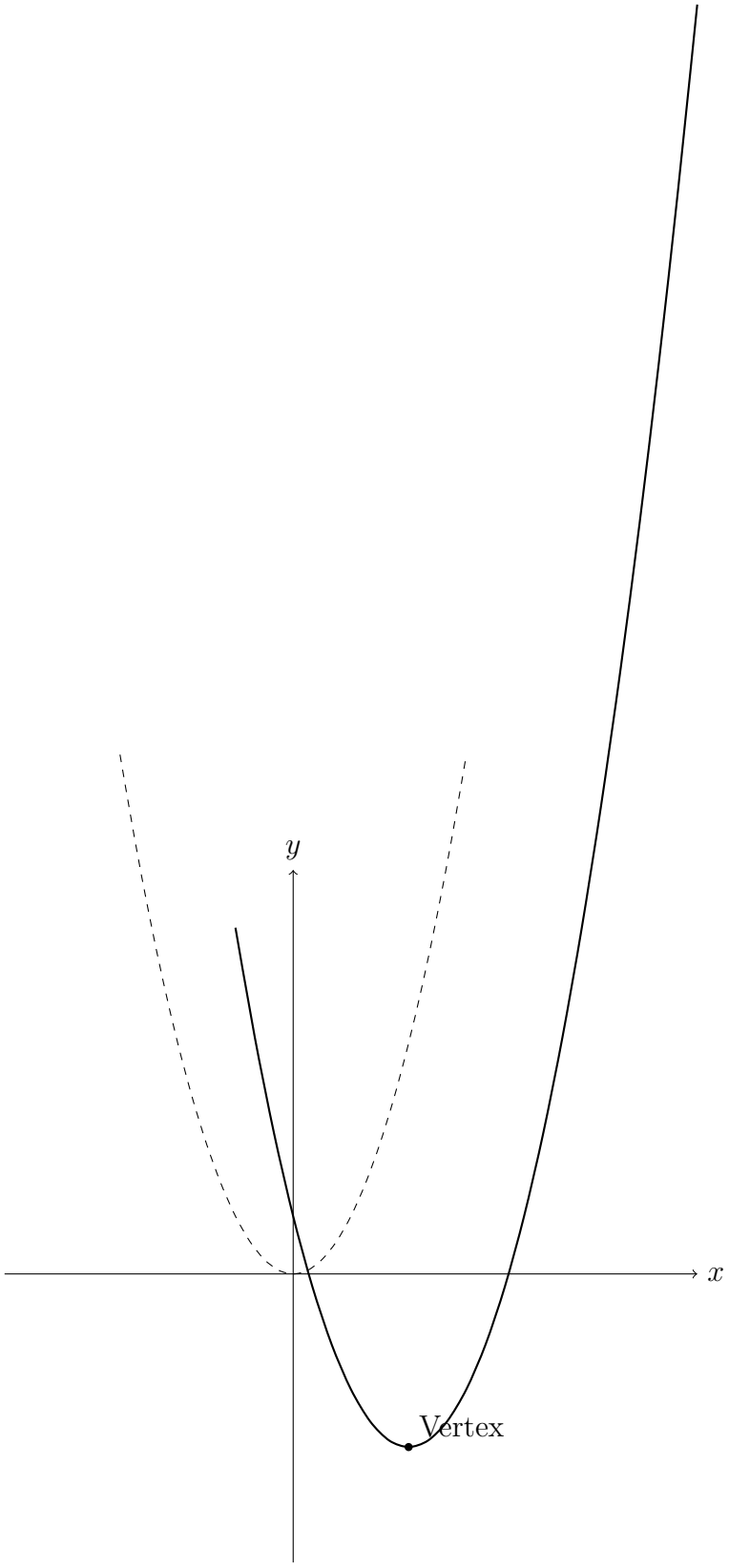
21. For
$$f(x) = x^2,$$
determine algebraically the function
$$g(x)$$
that results from applying: a vertical stretch by a factor of
$$2,$$
a reflection over the x-axis, a horizontal translation
$$3$$
units to the right, and a vertical translation
$$5$$
units down.

22. Starting with
$$f(x) = x^3,$$
derive the expression for
$$g(x)$$
when the following transformations are applied in order: a horizontal reflection, a horizontal compression by a factor of
$$\frac{1}{2},$$
a vertical stretch by a factor of
$$4,$$
and a translation
$$2$$
units to the left.

23. Given
$$f(x) = \sqrt{x},$$
show the complete sequence of transformations that produces
$$g(x) = 2\sqrt{5 - x} - 1.$$

24. For
$$f(x) = |x|,$$
determine algebraically the function
$$g(x)$$
obtained when a vertical stretch by
$$3,$$
a horizontal shift left by
$$4,$$
and a reflection in the x-axis are applied.

25. Sketch the graphs of
$$f(x) = x^2$$
and
$$g(x) = (x - 2)^2 - 3$$
on the same set of axes. Use the diagram provided to plot both curves (the original parabola should be drawn dashed and the transformed parabola solid) and label the vertex of
$$g(x).$$



26. For

$$f(x) = x^2,$$

determine the function

$$g(x) = -\frac{1}{2}\left(\frac{x+1}{3}\right)^2 + 2$$

by explaining each transformation (reflection, horizontal stretch, vertical scaling and translation) that has been applied.

27. Given

$$g(x) = 4 - 3\left(\frac{x-2}{5}\right)^2,$$

list the sequence of transformations that take

$$f(x) = x^2$$

to

$$g(x),$$

and explain the effect of each step.

28. For

$$f(x) = x^2$$

, the function

$$g(x) = 2(x+3)^2 - 10$$

is obtained by a series of transformations. Determine the image of the point

$$(0, 0)$$

under these transformations.

29. For

$$f(x) = x^2,$$

consider the transformation

$$g(x) = -4\left(\frac{x+2}{3}\right)^2 + 7.$$

List in order the transformations applied and find the coordinates of the vertex as well as one additional point on

$$g(x).$$

30. A function

$$g(x)$$

is defined in the form

$$g(x) = m \cdot f\big(n(x-p)\big) + q,$$

where

$$f(x) = x^2.$$

If

$$g(x) = -3\Big[2(x-5)\Big]^2 + 6,$$

determine the values of

$$m,$$

$$n,$$

$$p,$$

and

$$q.$$

Then, describe in order the sequence of transformations applied to

$$f(x).$$