ILLAWARRA TUTORING

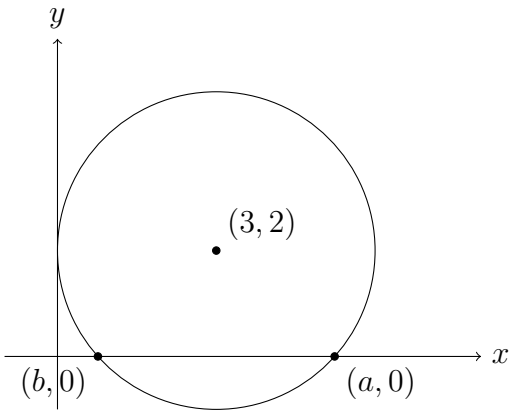
In this worksheet you will discover how to write equations for circles and semicircles and interpret their graphical representations.

Easy Questions

1. Write the equation of a circle with centre $(2, 3)$ and radius 5.
2. Determine the centre and radius of the circle $(x-4)^2 + (y+1)^2 = 16$.
3. Plot on pen and paper the circle $(x+2)^2 + (y-3)^2 = 9$. Make sure to label the centre and mark the radius.
4. Write the equation for the upper semicircle (i.e. the part where $y \geq 0$ assuming the centre is at the origin) of a circle with centre $(0, 0)$ and radius 4.
5. Identify the domain and range of the upper semicircle given by the equation $y = \sqrt{16 - x^2}$.

Intermediate Questions

11. Given that the endpoints of a diameter of a circle are $(-3, 4)$ and $(5, 4)$, find the equation of the circle.
12. A circle passes through $(2, 6)$ and has centre $(2, 2)$. Write its equation.
13. Rewrite the equation $x^2 + y^2 + 4x - 6y + 9 = 0$ in standard form.
14. Solve for y in terms of x for the circle $(x-1)^2 + (y+2)^2 = 25$.
15. Determine the points where the circle $(x-3)^2 + (y-2)^2 = 9$ intersects the x-axis.



16. Write the equation of a circle with centre $(-1, 5)$ that passes through $(3, 5)$.

17. Calculate the area and circumference of the circle $(x-2)^2 + (y+3)^2 = 36$.

18. Write the equation of the lower semicircle of the circle $(x+4)^2 + (y-1)^2 = 49$.

19. Find the intersection points of the circle $x^2 + y^2 = 25$ and the line $y = 3$.

20. Calculate the length of the chord of the circle $(x-1)^2 + (y+1)^2 = 20$ cut off by the line $y = 2$.

21. A circle is tangent to the x-axis and has its centre on the vertical line $x = 4$. If it passes through $(4, 7)$, find its equation.

22. Find the endpoints of a diameter of the circle $x^2 + (y-3)^2 = 16$ that is parallel to the x-axis.

23. Identify the centre and radius of the circle given by $x^2 + y^2 - 8x + 10y + 4 = 0$.

24. Convert the general form $x^2 + y^2 + 2x - 4y - 11 = 0$ to standard form.

25. Write the equation for the semicircular region (upper half) of the circle $(x-2)^2 + (y+2)^2 = 25$ where $y \geq -2$.

Hard Questions

21. Derive the general equation of a circle with centre (h, k) and radius r. Explain the meaning of each term in your answer.

22. Prove that the line $y = mx + c$ is tangent to the circle $(x-a)^2 + (y-b)^2 = r^2$ when the perpendicular distance from the centre (a, b) to the line equals r. Then, using this result, find the point of tangency when $m = 2$, $c = -1$ and the circle is $(x-3)^2 + (y+4)^2 = 25$.

23. Determine the equation of a circle that passes through the three non-collinear points $(1, 2)$, $(4, 6)$ and $(5, 2)$.

24. Find the equation of a circle that has its centre on the line $y = x$ and is tangent to the line $y = 0$, given that it passes through $(3, 4)$.

25. For a circle given by $x^2 + y^2 + Dx + Ey + F = 0$, outline the steps required to complete the square in order to find its centre and radius. Apply your method to the equation $x^2 + y^2 - 6x + 8y - 11 = 0$.

26. The line $y = 3$ cuts the circle $(x-2)^2 + (y-3)^2 = 20$ into a segment. Determine the area of this segment.

27. Find the equation of a circle that is tangent to the line $x = 0$ and passes through the point $(4, 5)$, with its centre located in the first quadrant.

28. Convert the circle $x^2 + y^2 - 4x + 2y - 20 = 0$ into standard form and state the equation for its upper semicircle.

29. Prove that for a circle the distance d from the centre to a chord of length L is given by $d = \sqrt{r^2 - \left(\frac{L}{2}\right)^2}$. Then, using this result, find the distance from the centre of the circle $(x-1)^2 + (y+2)^2 = 50$ to the chord defined by the endpoints where the circle intersects the line $y = 2$.

30. A semicircle is defined as the upper half of a circle. Find the radius of the circle such that the area of its upper semicircle equals the area of a rectangle with length equal to the diameter of the circle and width equal to 1.