ILLAWARRA TUTORING

In this worksheet you will discover how to write equations for circles and semicircles and interpret their graphical representations.

Easy Questions

1. Write the equation for a circle with centre $(0, 0)$ and radius 3.
2. Identify the centre and radius of the circle given by $(x-2)^2 + (y+5)^2 = 16$.
3. Write the equation for the upper semicircle of a circle with centre $(0, 0)$ and radius 4. (Give the domain of x.)
4. Given the equation $(x+3)^2 + (y-2)^2 = 25$, state the centre and the radius of the circle.
5. Determine whether the point $(5, 3)$ lies on the circle $(x-2)^2 + (y-3)^2 = 9$.

Intermediate Questions

6. Complete the square to express $x^2 + y^2 + 4x - 6y + 9 = 0$ in standard form and state its centre and radius.
7. Find the centre and radius of the circle with equation $x^2 + y^2 - 8x - 10y + 21 = 0$.
8. The endpoints of a diameter of a circle are $(1, 2)$ and $(7, 8)$. Find the centre, radius, and the equation of the circle.
9. Find the x-intercepts of the circle $(x-3)^2 + (y-4)^2 = 25$.
10. Write the equation for the semicircle with centre $(0, 0)$ and radius 5 that lies to the right of the y-axis. (Specify the appropriate domain.)
11. A circle is defined by $(x-6)^2 + (y+1)^2 = 49$. Find the coordinates of the point on this circle having the maximum y-coordinate.
12. Transform $x^2 + y^2 - 8x + 6y + 9 = 0$ to standard form by completing the square and state the centre and radius.
13. The endpoints of a diameter of a circle are $(-3, 2)$ and $(5, 2)$. Write the equation of the circle.
14. Derive the equation for a circle that is tangent to the x-axis, has its centre on the line $y = 5$, and passes through the point $(3, 9)$. (There may be more than one solution.)

15. Write the equation of the semicircle that has its diameter endpoints at $(-2, 0)$ and $(6, 0)$ and lies above the x-axis.

16. A circle has its centre at $(2, -3)$ and an area of 9π. Write its equation.

17. Find the equation of the circle whose centre lies on the y-axis and that passes through the points $(4, 2)$ and $(-4, 2)$.

18. Find the intersection points of the circle $(x-1)^2 + (y+2)^2 = 10$ with the y-axis.

19. A circle with centre $(0, 0)$ is tangent to the line $y = \sqrt{3}\,x - 2$. Find the radius of the circle.

20. Find the equation of the circle that passes through the points $(1, 2)$, $(3, 2)$, and $(3, 4)$.

Hard Questions

21. Prove that the general equation $x^2 + y^2 + Dx + Ey + F = 0$ represents a circle provided that $D^2 + E^2 - 4F > 0$. Then, show that the centre is $\left(-\frac{D}{2}, -\frac{E}{2}\right)$ and the radius is $\sqrt{\frac{D^2 + E^2 - 4F}{4}}$.

22. A circle and a semicircle share the same centre. The circle has radius 7 and the semicircle has radius 5, with the semicircle lying in the upper half-plane. Write the equations for both shapes and calculate the area of the region that lies inside the circle but outside the semicircle.

23. Find the length of the chord of the circle $(x-4)^2 + (y+1)^2 = 25$ that is cut by the horizontal line $y = -1$.

24. Determine the equation of the circle that has the same centre as the circle $(x-2)^2 + (y+3)^2 = 9$ and which is externally tangent to it with a radius that is 5 greater.

25. A circle of radius 6 is reduced to a semicircle by removing its lower half. Write the equation for the remaining region (the upper semicircle) and calculate its perimeter (include the length of the straight edge).

26. Find the coordinates of the intersection points of the circle $(x-3)^2 + (y-2)^2 = 25$ and the line passing through $(3, 2)$ with slope 2.

27. A semicircle is defined by the equation $y = \sqrt{49 - (x-2)^2}$ for $x \in [-5, 9]$. Determine the centre, radius, and the area of the semicircle.

28. Show that a point (a, b) lies on a circle with centre (h, k) and radius r if and only if $(a-h)^2 + (b-k)^2 = r^2$. Use this result to verify that the point $(5, 2)$ lies on the circle $(x-3)^2 + (y+1)^2 = 13$.

29. Find the equation of a circle whose centre lies on the line $x = 1$ and that is tangent to both the circle $(x-1)^2 + (y-1)^2 = 4$ and the x-axis.

30. Find the equation of the circle that passes through the points $(-1, 2)$, $(3, 4)$, and $(5, -2)$. (Hint: Write the equation in the form $x^2 + y^2 + Dx + Ey + F = 0$ and solve for D, E, and F.)