ILLAWARRA TUTORING

This worksheet will help you learn to work with absolute value functions and understand how their graphs are formed. You will be required to evaluate expressions, rewrite absolute value functions in piecewise form, solve equations and inequalities, and describe transformations. Work carefully and show all your steps.

Easy Questions

1. Evaluate the following expressions: Write your answers in simplest form.
2. Sketch the graph of the function given by $f(x) = |x|$.
3. Solve the equation $|x| = 5$.
4. Write the function $f(x) = |x - 2|$ in piecewise form.
5. Determine the vertex of the function $f(x) = |x + 3|$.

Intermediate Questions

6. Solve the equation $|2x - 4| = 6$.
7. Solve the inequality $|x + 1| < 4$.
8. Sketch the graph of $f(x) = 2|x - 1| - 3$.
9. Write the function $f(x) = 3|x + 2| - 5$ in piecewise form.
10. Solve the equation $|x - 3| + |x + 1| = 5$.
11. Find the points of intersection of the graphs $y = |x|$ and $y = 4 - x$.
12. Solve the equation $|x| = |2x - 1|$.
13. Describe the transformation that converts the function $y = |x|$ into $y = |x - 2| + 3$.
14. Give the equation of the function obtained by shifting $y = |x|$ right by 2 and up by 3.
15. Solve the inequality $|3x + 2| > 5$.
16. Solve the equation $|2x - 3| = |x + 4|$.
17. Solve for x in the equation $|x - 1| = |2 - x|$.

18. Express the function $f(x) = |x - 2| + 2$ in piecewise form and sketch its graph.

19. Solve the equation $|x - 1| - |x + 2| = 0$.

20. Solve the inequality $|2x + 3| \leq |x - 1|$.

Hard Questions

21. Derive the piecewise form of the function $f(x) = |2x + 4| - |x - 1|$.

22. Solve the equation $|x - 2| - |2x + 3| = 1$.

23. Find all solutions of the equation $|x + 3| - 2|x - 1| = 5$.

24. For the function $f(x) = 2|x + 1| - 3|x - 2|$, determine the intervals where the function is linear and find the corresponding expressions in each interval.

25. If $f(x) = |x - 3|$ and $g(x) = |2x + 1|$, find the x-coordinates of their points of intersection.

26. Solve the inequality $|3x - 2| - |x + 4| < 0$.

27. Prove that the equation $|x - 1| + |2x + 3| = |3x + 2|$ has at most two solutions and find those solutions.

28. Consider the function $f(x) = |x - a|$, where a is a real number. Describe how the graph changes as a varies. Illustrate your answer by describing the graphs when $a = 2$ and when $a = -3$.

29. Find the range of the function $f(x) = 2|x + 1| - |x - 3|$.

30. For the function $f(x) = |x - 2| + |x + 4|$, determine the minimum value and the value(s) of x at which this minimum occurs.