



In this worksheet you will use the discriminant, given by  $D = b^2 - 4ac$ , to determine the nature of the roots of quadratic equations. You will practice calculating the discriminant and explaining what its value means about the solutions.

## Easy Questions

1. Identify the coefficients in the quadratic equation  $2x^2 + 3x - 5 = 0$ . Write down the values of  $a$ ,  $b$ , and  $c$ .
2. Calculate the discriminant of  $x^2 + 4x + 3 = 0$ . What does its value indicate about the roots?
3. Find the discriminant of  $2x^2 - 6x + 4 = 0$ , and state whether the equation has real roots.
4. Determine the nature of the roots of  $x^2 - 4x + 4 = 0$  by calculating its discriminant.
5. Use the discriminant to determine whether  $3x^2 + 2x + 1 = 0$  has any real roots.

## Intermediate Questions

6. Calculate the discriminant of  $2x^2 - 4x + 1 = 0$ . Based on your calculation, state whether the roots are real and distinct, equal, or complex.
7. Determine all values of  $k$  for which the quadratic equation  $x^2 + kx + 9 = 0$  has two distinct real roots.
8. Find the value(s) of  $k$  such that  $3x^2 - kx + 12 = 0$  has equal (repeated) real roots.
9. For the quadratic equation  $mx^2 + 10x + m = 0$ , find the range of values of  $m$  for which the equation yields no real roots.
10. Compute the discriminant of  $5x^2 + 4x + 1 = 0$  and state the nature of its roots.
11. Explain why a quadratic function with a negative discriminant does not intersect the  $x$ -axis. Support your explanation with reference to the formula  $D = b^2 - 4ac$ .
12. Given  $7x^2 - 2x + c = 0$ , determine the range of values of  $c$  that ensure the equation has real roots.
13. Rewrite  $(x - 3)^2 = 0$  in standard form and compute its discriminant.

14. For the quadratic equation  $x^2 + bx + 4 = 0$ , if the discriminant equals zero, determine the value(s) of  $b$ .
15. For  $2x^2 - 3x + k = 0$ , find the range of  $k$  for which the equation has two distinct real solutions.
16. Determine whether the quadratic equation  $6x^2 + 7x + 2 = 0$  has rational roots by computing its discriminant.
17. Compute the discriminant of  $4x^2 + 12x + 9 = 0$  and explain what this value implies about the roots.
18. Find the values of  $p$  for which the equation  $x^2 + 2px + p^2 - 4 = 0$  has real roots. Explain your reasoning.
19. Using the discriminant, determine the number and type of roots of  $9x^2 + 24x + 16 = 0$ .
20. For  $8x^2 - x + c = 0$ , find the value of  $c$  that makes the quadratic have a repeated (double) root.

## Hard Questions

21. Prove that if a quadratic function  $ax^2 + bx + c$  has  $D = 0$ , then its vertex lies on the  $x$ -axis.
22. Derive the condition on  $a$ ,  $b$ , and  $c$  for which the graph of  $ax^2 + bx + c = 0$  is tangent to the  $x$ -axis.
23. For the equation  $kx^2 + 2(k - 1)x + 3 = 0$ , determine the values of  $k$  that result in real roots.
24. The quadratic equation  $x^2 + (m - 2)x + m = 0$  has equal roots. Determine the value of  $m$ .
25. Find the range of  $c$  for which the quadratic function  $f(x) = 2x^2 + 4x + c$  does not intersect the  $x$ -axis.
26. Determine the condition on  $k$  such that the quadratic equation  $(k + 1)x^2 - 2kx + k - 3 = 0$  has two distinct real roots.
27. A quadratic equation has roots in the ratio 3 : 1 and a discriminant equal to 64. Find the quadratic equation.
28. Suppose the roots of  $x^2 + bx + c = 0$  are such that one root is twice the other and the discriminant is 36. Express  $b$  and  $c$  in terms of the smaller root.
29. The coefficients of a quadratic equation are  $a$ ,  $a + d$ , and  $a + 2d$ , where  $d \neq 0$ . Show that its discriminant is  $D = -3a^2 - 6ad + d^2$  and deduce the condition on  $a$  and  $d$  for the equation to have real roots.
30. For a quadratic equation  $ax^2 + bx + c = 0$  with a fixed discriminant  $D$ , explain how increasing the coefficient  $a$  (while keeping  $b$  constant) affects the value of  $c$  and the position of the vertex with respect to the  $x$ -axis.