In this worksheet you will learn how to use the discriminant of a quadratic equation, given by $D = b^2 - 4ac$, to determine the nature of its roots. Follow the instructions for each question and show all your working.

Easy Questions

1. Calculate the discriminant of the quadratic equation $2x^2 + 3x + 1 = 0$.

2. Calculate the discriminant of the quadratic equation $x^2 + 2x + 5 = 0$ and state the nature of its roots.

3. Find the discriminant of $x^2 + 6x + 9 = 0$ and explain what this indicates about the roots.

4. Determine the discriminant of $3x^2 - 4x + 2 = 0$ and state whether the roots are real or complex.

5. In one sentence, explain the significance of the discriminant in determining the nature of the roots of a quadratic equation.

Intermediate Questions

6. For the quadratic equation $5x^2 - 20x + 15 = 0$, compute the discriminant and state the nature of its roots.

7. Compute the discriminant of $2x^2 - 4x + 2 = 0$ and state your conclusion about the roots.

8. Determine the condition on the parameter m for which the quadratic $x^2 + mx + 1 = 0$ has real roots.

9. For the equation $x^2 - 4x + c = 0$, find the values of c for which the equation has non-real roots.

10. Compute the discriminant of $-x^2 + 4x - 4 = 0$ and describe the nature of its roots.

11. For the quadratic equation $3x^2 + (k - 2)x + 1 = 0$, determine the range of values for k such that the equation has real roots.

12. Given $x^2 - (2p + 3)x + p^2 + 3p + 2 = 0$, show that the discriminant is a perfect square and state its value.

13. Verify that the quadratic $2x^2 - 4x + 2 = 0$ has a discriminant of zero and explain what this signifies for its roots.

14. Explain in a short paragraph why a quadratic equation $ax^2 + bx + c = 0$ with $D < 0$ has no roots, based on the quadratic formula.

15. Find the discriminant of $7x^2 + 2x - 3 = 0$ and indicate whether the roots are real and distinct, equal or non-real.

16. For $x^2 + px + q = 0$, if the discriminant is zero, show that $q = \dfrac{p^2}{4}$.

17. In the quadratic $kx^2 - 6x + 9 = 0$, determine the value of k for which the equation has a repeated (double) root.

18. Determine the discriminant of $4x^2 - 4x + 1 = 0$ and state the nature of the roots.

19. Explain in a few sentences what it means for a quadratic equation to have a discriminant of zero regarding its solutions.

20. For the quadratic $x^2 + mx + 9 = 0$, find the range of values of m for which the equation has no real roots.

Hard Questions

21. Show that if a quadratic equation $ax^2 + bx + c = 0$ has a negative discriminant, then its graph does not cross the x-axis. Provide a brief explanation using the quadratic formula.

22. If the quadratic $mx^2 + 4x + 1 = 0$ has equal roots, determine the value of m.

23. The difference between the roots of $x^2 + px + q = 0$ is 1. Show that $p^2 = 1 + 4q$, and hence deduce the discriminant.

24. For a quadratic $ax^2 + bx + c = 0$, explain how an increase in the discriminant affects the distance between the two real roots. Justify your answer.

25. For the quadratic equation $x^2 - 2kx + k^2 - 1 = 0$, determine the range of k for which the equation has real and distinct roots.

26. Consider the equation $(k+1)x^2 + 2(k+1)x + (k+1) = 0$. Calculate the discriminant and discuss the nature of the roots for different values of k, including the degenerate case.

27. For the quadratic $ax^2 + (a + 1)x + 1 = 0$, where $a \neq 0$, determine the condition on a under which the equation does not have two distinct real roots.

28. Suppose $ax^2 + bx + c = 0$ has a double root. Show that it can be written in the form $a(x + d)^2 = 0$ and express d in terms of a and b.

29. Find the discriminant of $3x^2 + bx + 4 = 0$ and determine the values of b for which the equation has no real roots.

30. A quadratic equation with leading coefficient 1 has roots that sum to s and have product p. Express its discriminant in terms of s and p, and deduce the inequality that must hold between s and p for the roots to be real.