ILLAWARRA TUTORING

In this worksheet you will study the properties and graphs of quadratic functions and explore how they model real-world phenomena. You will work on identifying key features of quadratics, transforming between forms, solving equations and applying these ideas to practical problems.

Easy Questions

1. The quadratic function is given by $f(x) = ax^2 + bx + c$. Identify the coefficients a, b and c in the function $f(x) = 2x^2 - 3x + 5$.
2. Determine the y-intercept of the quadratic function $f(x) = x^2 + 4x - 7$.
3. Calculate the value of the function $f(x) = x^2 - 2x + 1$ when $x = 3$.
4. State whether the graph of $f(x) = -3x^2 + 2x + 1$ opens upward or downward and briefly explain your reasoning.
5. Factor the quadratic expression $x^2 - 5x + 6$ completely.

Intermediate Questions

6. Rewrite $f(x) = x^2 + 4x + 3$ in vertex form by completing the square.
7. For $f(x) = -2x^2+8x-3$, determine the coordinates of the vertex and state whether the vertex represents a maximum or a minimum value.
8. Sketch the graph of $f(x) = x^2 - 6x + 8$. Include the intercepts and the vertex in your sketch.
9. Find the x-intercepts of $f(x) = x^2 - 5x + 6$ by factoring the quadratic.
10. Determine the y-intercept of $f(x) = 3x^2 + 2x - 4$.
11. Compute $f(1)$ for the function $f(x) = -x^2 + 3x + 2$.
12. Explain how changing the coefficient a in $f(x) = ax^2$ affects the width of the parabola. Provide an example to support your explanation.
13. Solve for x in the equation $x^2 - 4 = 0$.
14. A ball is thrown upward and its height (in metres) after t seconds is given by $h(t) = -5t^2 + 20t + 2$. Determine the time at which the ball reaches its maximum height.

15. Using your answer from the previous question, calculate the maximum height reached by the ball.

16. Find a quadratic function with x-intercepts 3 and 7 and a y-intercept of $(0, -21)$. Write your answer in factored form.

17. A quadratic function $f(x) = ax^2 + bx + c$ has a minimum value of 2 at $x = -1$ and $f(0) = 6$. Find the values of a, b and c.

18. Determine the value of $f(-1)$ for $f(x) = -x^2 + 2x + 3$.

19. Complete the following table for $f(x) = x^2 - 2x - 3$ and then sketch the graph of the function on graph paper.

x	$f(x)$
-2	
-1	
0	
1	
2	

20. Describe the effect on the graph of a quadratic function when a constant is added to $f(x)$ (i.e. $f(x) + k$). Support your explanation with an example.

Hard Questions

21. A ball is thrown from ground level with an initial velocity of 15 m/s. Its height is given by $h(t) = -4.9t^2 + 15t$. Determine the time when the ball hits the ground by solving $h(t) = 0$.

22. A garden arch is designed so that its shape is a parabola. The arch has a maximum height of 5 m at its vertex and meets the ground 4 m to the left and right of the vertex. Form a quadratic function that models the arch.

23. Given $f(x) = 2(x - 3)^2 - 7$, first expand the expression to standard form and then determine the x- and y-intercepts of the function.

24. Derive the formula for the x-coordinate of the vertex of a quadratic function in standard form, $f(x) = ax^2 + bx + c$.

25. If a quadratic function with roots p and q is written as $f(x) = a(x - p)(x - q)$, explain how this form demonstrates that the graph of the function is symmetric.

26. For the quadratic function $f(x) = -3x^2 + 6x + 9$, determine the coordinates of the vertex in fractional form without using a calculator.

27. Prove that for any quadratic function, the x-coordinate of the vertex is equal to the average of the two roots.

28. Solve the quadratic equation $5x^2 - 20x + 15 = 0$ by completing the square, showing all your working.

29. The area, A, of a rectangular field with a fixed perimeter of 60 m is given by $A(x) = x(30 - x)$, where x is the length of one side. Explain why $A(x)$ is a quadratic function and determine the maximum possible area.

30. Given that a quadratic function $f(x) = ax^2 + bx + c$ satisfies $f(1) = 3$, $f(2) = 5$ and $f(3) = 9$, determine the values of a, b and c by setting up and solving a system of equations.