ILLAWARRA TUTORING

This worksheet will explore the properties and graphs of quadratic functions. You will investigate standard and vertex forms, study the axis of symmetry and vertex, and see how quadratics can model real-world phenomena.

Easy Questions

1. Write the function $f(x) = x^2$ in standard form.
2. Find the axis of symmetry of the function $f(x) = x^2$.
3. Determine the vertex of $f(x) = x^2 + 4x + 4$.
4. Plot the graph of $f(x) = x^2$ on a set of axes. Use pen and paper to show the parabola and clearly mark the vertex.
5. Evaluate $f(2)$ for $f(x) = x^2 - 2x + 1$.

Intermediate Questions

6. Rewrite $f(x) = 2x^2 + 8x + 5$ in the form $f(x) = a(x - h)^2 + k$ by completing the square.
7. Find the x-intercepts of $f(x) = x^2 - 5x + 6$.
8. State whether the graph of $f(x) = -x^2 + 3x - 1$ opens upwards or downwards and explain why.
9. Find the vertex of $f(x) = -3x^2 + 6x - 2$.
10. Determine the y-intercept of $f(x) = 2x^2 - 3x + 1$.
11. For $f(x) = (x - 2)^2 + 3$, state the vertex and the axis of symmetry.
12. A ball thrown upwards is modelled by $h(t) = -5t^2 + 20t + 2$. Find the time at which the ball reaches its maximum height and determine that maximum height.
13. Determine the vertex of $f(x) = x^2 + 2x - 8$ by completing the square.
14. For $f(x) = 4x^2 - 12x + 9$, find the value of x at which the function attains its minimum or maximum, and state that extreme value.
15. Find the maximum value of $f(x) = -x^2 + 4x + 1$ by determining its vertex.

16. Write a quadratic function that has a vertex at $(2, -3)$ and passes through the point $(4, 5)$.

17. Determine the axis of symmetry for $f(x) = 3(x + 1)^2 - 7$.

18. Find the coordinates of the vertex for $f(x) = 5 - 2(x - 3)^2$.

19. Describe in words the key features (vertex, x-intercepts, and axis of symmetry) required to sketch the graph of $f(x) = -2(x-1)^2+4$. Then, use these descriptions to produce a rough sketch on paper.

20. Explain how the graph of $f(x) = x^2$ changes when the coefficient of x^2 is increased from 1 to 3.

Hard Questions

21. Derive the formula for the axis of symmetry for a quadratic function in the form $f(x) = ax^2 + bx + c$. Provide a step-by-step explanation.

22. Given a quadratic function $f(x) = ax^2 + bx + c$ with vertex (h, k), demonstrate how to rewrite the equation in the form $f(x) = a(x - h)^2 + k$. Include all necessary algebraic steps.

23. Describe a real-life situation that can be modelled using a quadratic function. Identify the variable representing time (or another independent variable), and explain what the vertex and x-intercepts would represent in your scenario.

24. A quadratic function has a maximum at $(3, 10)$ and passes through $(5, 2)$. Find the equation of the quadratic function in vertex form.

25. Solve for the vertex form by completing the square for $f(x) = 3x^2 + 12x + 7$, and hence determine the vertex.

26. Consider a quadratic function modelling a projectile given by $f(x) = A(x - p)^2 + q$. Explain the physical meaning of the parameters A, p, and q in terms of the projectile's motion.

27. For $f(x) = -2x^2 + 8x - 3$, determine if there is any change in concavity along the graph. Explain your reasoning.

28. Explain how vertical translations, horizontal translations, and reflections affect the graph of $f(x) = x^2$. Provide specific examples to illustrate each transformation.

29. Prove that the line passing through the vertex and parallel to the axis of symmetry divides the parabola $f(x) = ax^2 + bx + c$ into two congruent halves. Outline your geometric reasoning.

30. A company's profit is modelled by $f(x) = -x^2 + 40x - 300$, where x represents the number of units sold. Determine the value of x which produces the maximum profit and compute the maximum profit.