ILLAWARRA TUTORING

In this worksheet you will study the properties and graphs of quadratic functions to see how they model real-world phenomena. You will explore algebraic manipulation, factorisation, graph sketching and the interpretation of quadratic models.

Easy Questions

1. Identify the coefficients in the quadratic function $f(x) = 3x^2 + 2x - 5$. Write down the values of a, b and c.
2. For the quadratic function $f(x) = x^2 - 4x + 3$, state its y-intercept.
3. Evaluate the function $f(x) = 2x^2 - x + 1$ at $x = 2$. Show your calculation.
4. Sketch the graph of the function $f(x) = x^2$. Label the vertex and the y-intercept.
5. State whether the graph of $f(x) = x^2$ is symmetric. If yes, state the line of symmetry in words.

Intermediate Questions

6. Factorise $x^2 + 5x + 6$.
7. Solve for x in the equation $x^2 - x - 6 = 0$.
8. Rewrite $x^2 + 6x + 5$ in the form $(x + p)^2 + q$ by completing the square.
9. Find the x-intercepts of the function $f(x) = x^2 - 9$. Show all steps.
10. Sketch the graph of $f(x) = -x^2 + 4$ on the coordinate plane. Use the diagram below as a reference and mark the intercepts.
11. For the function $f(x) = -x^2 + 2x + 3$, state whether the parabola opens upwards or downwards and explain your reasoning based on the coefficient of x^2.
12. Determine the x-intercepts of $f(x) = x^2 - 9$ by expressing the quadratic in factorised form.
13. Explain the effect of changing the coefficient a in $f(x) = ax^2$ on the shape of the graph. Compare the cases when $a = 1$ and when $a = 2$.
14. Sketch the graph of $f(x) = \frac{1}{2}x^2 - 2$ on the provided coordinate grid. Clearly label important points such as the vertex and intercepts.

15. Discuss how quadratic functions can model real-world phenomena such as projectile motion. Describe one scenario and explain which features of the quadratic function are relevant.

16. For the function $f(x) = (x-1)^2 + 3$, determine the minimum value of f and the value of x at which this minimum occurs.

17. Solve the equation $(x+2)^2 = 16$ for x. Show all steps.

18. Find a quadratic function in the form $f(x) = a(x-r_1)(x-r_2)$ that has x-intercepts at $x = 2$ and $x = 5$ and a y-intercept of 10. Explain your reasoning.

19. Describe the effect of translating the quadratic function $f(x) = x^2$ two units to the left and three units down. Write the equation of the new function.

20. Given $f(x) = 3x^2 + 6x + c$ and that $f(0) = 9$, determine the value of c.

Hard Questions

21. Derive the formula for the vertex of a quadratic function in the form $f(x) = ax^2 + bx + c$ by completing the square. Provide all steps in your derivation.

22. For a quadratic function $f(x) = ax^2 + bx + c$, discuss in detail how variations in the coefficient b affect the position of the vertex. Justify your explanation.

23. For the function $f(x) = -2x^2 + 4x + 1$, determine the intervals on which f is increasing and the intervals on which f is decreasing. Explain your reasoning.

24. If a quadratic function $f(x) = ax^2 + bx + c$ is known to have a maximum value, what can be concluded about the sign of a? Provide a justification for your answer.

25. A ball is thrown and its height h (in metres) at time t (in seconds) is modelled by a quadratic function. If the ball reaches its highest point at $t = 2$ seconds and lands at $t = 5$ seconds, outline the steps you would take to formulate a quadratic function modelling this scenario. Do not perform any calculations.

26. Explain how the graph of a quadratic function is related to its factorised form $f(x) = a(x-r_1)(x-r_2)$. In your answer, discuss the significance of r_1 and r_2.

27. For the quadratic function $f(x) = a(x-h)^2 + k$, discuss the effect of increasing h and k on the graph of f. Illustrate your explanation with a diagram.

28. Establish and explain the relationship between the coefficients in $f(x) = ax^2+bx+c$ and the position of its x-intercepts (when they exist).

29. Consider the real-world example of designing a parabolic arch. Explain how quadratic functions are utilised in the design process, including how altering the coefficients can change the shape and dimensions of the arch.

30. For the quadratic function $f(x) = \frac{1}{2}x^2 - 4x + 7$, determine all intercepts and sketch an accurate graph of the function. On your sketch, label the x-intercepts, the y-intercept and the vertex.