ILLAWARRA TUTORING

In this worksheet you will explore key characteristics of functions. You will investigate continuity, increasing/decreasing behaviour and symmetry so that you can better understand how functions behave.

Easy Questions

1. Write a short definition of what it means for a function to be continuous.
2. Write a short definition of what it means for a function to be increasing.
3. For the function $f(x) = x^2$, state whether the function is even, odd or neither.
4. For the function $f(x) = |x|$, state whether the function is even, odd or neither.
5. Answer true or false: The function $f(x) = 3$ is continuous everywhere.

Intermediate Questions

6. Consider the function $f(x) = \begin{cases} x+1 & \text{if } x < 0, \\ 2-x & \text{if } x \geq 0. \end{cases}$ Write a brief explanation stating whether the function is continuous at $x = 0$.
7. For $f(x) = \dfrac{x^2-1}{x-1}$, simplify the expression and explain whether f is continuous at $x = 1$.
8. For the function $f(x) = x^2$, state on which interval the function is decreasing and on which interval it is increasing.
9. Consider $f(x) = \dfrac{1}{x}$. Discuss its continuity and describe its behaviour on the intervals $x < 0$ and $x > 0$.
10. Given $f(x) = \begin{cases} x^2 & (x \leq 2), \\ 4x-4 & (x > 2), \end{cases}$ determine if f is continuous at $x = 2$ and describe if f is increasing or decreasing on the intervals defined by its pieces.
11. For the function $f(x) = \sqrt{x^2}$, state whether it is even, odd or neither.
12. Consider $f(x) = x^2 + 3$. Explain whether this function is continuous everywhere and state its symmetry.

13. For the function $f(x) = |2x - 4|$, discuss its continuity and identify any symmetry it may have.

14. In your own words, explain what it means for a function to be monotonically increasing. Provide an example of such a function.

15. Consider the constant function $f(x) = 7$. State whether this function is increasing, decreasing, or constant.

16. For $f(x) = \sqrt{x}$, determine if there is any line of symmetry in its graph.

17. Let $f(x) = \begin{cases} 2x + 1 & (x < 0), \\ x^2 & (x \geq 0). \end{cases}$ Determine whether f is continuous at $x = 0$ and briefly discuss its increasing/decreasing behaviour on the domains $x < 0$ and $x \geq 0$.

18. For the function $f(x) = x^4$, state whether it is even, odd or neither. Also, indicate the intervals on which it is increasing or decreasing.

19. Write a short example that explains what is meant by a function being discontinuous.

20. For the function $f(x) = x^2 + |x|$, discuss the continuity of f and determine whether it is even, odd or neither.

Hard Questions

21. Given $f(x) = \dfrac{x^2 - 4}{x - 2}$, identify all points where f is discontinuous. Explain your reasoning.

22. Explore the function $f(x) = \dfrac{x^2}{x^2 - 9}$. Discuss its continuity and describe any asymptotic behaviour. Also, indicate in words how the function behaves (increasing or decreasing) in intervals separated by its discontinuities.

23. Consider $f(x) = \sin(x)$. State whether the function is even, odd or neither and describe the intervals within which f is increasing or decreasing on the interval $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

24. Let $f(x) = x + \dfrac{1}{x}$ for $x \neq 0$. Identify all discontinuities and discuss any symmetry properties of f.

25. For the function $f(x) = \dfrac{x + 1}{x - 1}$, determine all points of discontinuity. Describe how the function behaves as x approaches the discontinuity and state any symmetry you observe.

26. Consider $f(x) = \sqrt{x + 4} - \sqrt{x}$. Determine the intervals on which f is increasing and provide a brief explanation concerning its continuity.

27. Suppose that f is a function defined on the entire real line that is continuous everywhere and is symmetric about the y-axis. Explain what properties f must have and justify your answer.

28. Discuss how the operations involving the absolute value can affect the continuity and increasing/decreasing behaviour of a function. Include specific examples in your explanation.

29. Let $f(x) = \dfrac{2x-5}{x+3}$. Identify any discontinuities and, using a sign chart or sample values, discuss the intervals where f is increasing or decreasing. Justify your reasoning without referring to calculus.

30. Write a summary explaining how the properties of continuity, increasing/decreasing behaviour and symmetry can assist you in sketching a rough graph of a function. Include at least one example in your explanation.