ILLAWARRA TUTORING

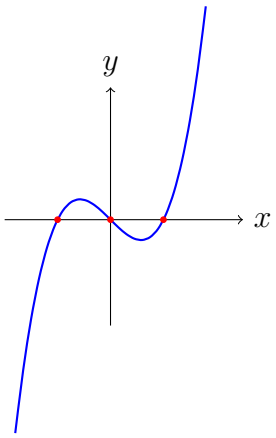
This worksheet on Polynomial functions will extend your understanding of higher-degree polynomials and their general behaviours. You will explore the structure, factorisation, zeros, and end behaviours of polynomials as well as work on proving fundamental properties. Do all questions on your own and show full working.

Easy Questions

1. Define a polynomial function and give one example.
2. For the polynomial $f(x) = 4x^5 - 3x^3 + 2x - 7$, identify its degree and leading coefficient.
3. Evaluate $f(x) = x^3 - 2x + 1$ at $x = 2$.
4. Write the expression $3 + 5x - 2x^2$ in standard form (i.e. descending powers of x).
5. In the polynomial $7x^4 - 3x^2 + x - 9$, identify the constant term.

Intermediate Questions

6. Factorise $x^3 + 3x^2 - x - 3$ completely.
7. Show that $x = 1$ is a root of $2x^3 - 5x^2 + x + 2$ and factorise the polynomial completely.
8. Determine the end behaviour of $f(x) = -x^4 + 3x^3 - 2x + 7$.
9. Sketch the graph of $f(x) = x^3 - x$ on a coordinate grid. Indicate its intercepts and overall shape. To assist, see the reference diagram below.)



10. Factorise completely and determine the zeros of $f(x) = 2x^3 - 6x^2 + 4x$.
11. Use the Remainder Theorem to find the remainder when $f(x) = x^4 - 2x^3 + 3x^2 - 4x + 5$ is divided by $(x - 1)$.
12. Explain why any polynomial function with an odd degree must have at least one real root.
13. Find all zeros of $f(x) = x^3 - 4x$.
14. Factorise $f(x) = x^4 - 5x^2 + 4$ completely.
15. Determine the coefficient of x^2 in the product $(x + 2)(x^2 - 3x + 4)$.
16. Expand and simplify $(2x - 3)(x^2 + x - 5)$.
17. Simplify $(x^2 - 2x + 1) - (x^2 + 3x - 4)$ and write the answer in standard form.
18. Determine whether $x = 2$ is a solution of $f(x) = x^3 - 3x^2 + 2x + 4$.
19. For a polynomial of degree 5, state the maximum possible number of turning points.
20. Identify the degree and the leading coefficient of $f(x) = -3x^5 + 2x^3 - x + 7$.

Hard Questions

21. Find all zeros of $f(x) = x^4 + 4$.
22. Use the Remainder Theorem to find the remainder when $f(x) = 2x^5 - 3x^4 + x^3 - 7x^2 + 5x - 2$ is divided by $(x - 2)$.
23. Use synthetic division to divide $f(x) = x^3 - 2x^2 - x + 2$ by $(x - 1)$ and state the quotient.
24. Prove that if a polynomial $f(x)$ has degree n, then it has at most n real roots.
25. Determine all values of a for which $f(x) = x^3 + ax^2 + a^2x + 1$ is divisible by $(x + 1)$.
26. The polynomial is given by $f(x) = (x - 2)^3 + 3(x - 2)^2 - 2(x - 2) + k$. Expand $f(x)$ and determine the value of k for which $f(2) = 0$.
27. Let $f(x) = (x - 1)^2(x + 2) + m$. Determine the value of m such that $x = 1$ is a double root of $f(x)$.
28. If $f(x) = (x + 2)(x - 3)(x^2 + 1)$, determine the number of real zeros of $f(x)$.
29. Using pen and paper, sketch the general shape of a quintic polynomial with a negative leading coefficient that has three distinct real zeros. Ensure your sketch clearly indicates the x-intercepts and turning points.
30. Prove that the sum of the coefficients of a polynomial $f(x)$ is equal to $f(1)$. Provide a detailed explanation and include an example.