ILLAWARRA TUTORING

This worksheet focuses on developing your understanding of higher-degree polynomials and their general behaviours. You will practise operations including addition, subtraction, multiplication, division, factorisation and analyse their characteristics such as degree, leading coefficients, roots and end behaviour.

Easy Questions

1. Simplify $(2x^2 + 3x - 4) + (5x^2 - x + 7)$.
2. Determine the degree and the leading coefficient of $4x^3 - 2x^2 + x - 6$.
3. Evaluate $P(x) = x^3 - 2x^2 + 3x + 5$ at $x = 2$.
4. Factor out the greatest common factor from $6x^4 - 9x^3 + 3x^2$.
5. Write the polynomial $3x^2 - 5$ in standard form and state its constant term.

Intermediate Questions

6. Simplify $(x^2 + 3x + 2) + (2x^2 - x - 5)$ to express the result in standard form.
7. Evaluate $(3x^3 + 2x^2 - x) - (x^3 - x^2 + 4)$ and simplify your answer.
8. Multiply $(x + 2)$ by $(x^2 - x + 3)$ and express the result in standard form.
9. Divide $6x^3 - 3x^2 + 9x$ by x and simplify your answer.
10. Determine whether $p(x) = x^4 - 2x^2 + 1$ is even, odd, or neither.
11. Expand $(x + 1)^3$ and state the coefficient of x^2.
12. For $f(x) = 2x^3 - 3x^2 + x + 5$, find $f(-1)$.
13. Factorise $x^3 - 3x^2 + 2x$ completely.
14. Sketch the graph of $f(x) = x^3 - 6x^2 + 9x$ on pen and paper. Briefly describe its turning points and end behaviour.
15. Find all real roots of $f(x) = x^4 - 5x^2 + 4$.
16. For $g(x) = 3x^3 - 6x^2 + x - 2$, determine the product of its roots.
17. Factorise $h(x) = x^3 + x^2 - x - 1$ by grouping.

18. State the maximum number of turning points that a polynomial of degree 5 can have.

19. Show that $(x-2)$ is a factor of $f(x) = x^3 - 4x^2 + 4x$, and then factorise $f(x)$ completely.

20. Determine the end behaviour of $f(x) = -2x^4 + 3x^3 - x + 7$.

Hard Questions

21. Prove that a non-zero polynomial of degree n has at most n real roots.

22. For $f(x) = x^5 - 5x^3 + 4x$, discuss its symmetry (odd or even) and use your conclusion to deduce possible factorisations.

23. Find all real roots of $f(x) = x^4 - 5x^2 + 4$ and state the multiplicity of each root.

24. Given $p(x) = 2x^4 - 3x^3 - 11x^2 + 12x + 9$, use synthetic division to divide by $(x+1)$ and, if possible, factorise $p(x)$ completely.

25. Determine the remainder when $f(x) = 3x^5 - 2x^4 + 6x^3 - x^2 + 4x - 5$ is divided by $(x-2)$.

26. Show that $p(x) = x^3 - 3x^2 + 3x - 1$ can be written as $(x-1)^3$, and explain the nature of the graph at $x = 1$.

27. Solve $x^4 - 10x^2 + 9 = 0$ by making an appropriate substitution.

28. Consider a general quartic polynomial $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ with $a > 0$. Discuss how the leading coefficient and degree influence the end behaviour of the graph.

29. For $f(x) = x^5 - 5x^4 + 5x^3 + 5x^2 - 6x + 1$, show that $(x-1)$ is a factor and factorise $f(x)$ completely.