In this worksheet you will extend your understanding to higher-degree polynomials and their general behaviours. You will explore evaluation, factoring, expanding, the Remainder and Factor Theorems, end behaviour and zero multiplicities. This practice prepares you for advanced topics by focussing on the structure and properties of polynomial functions.

Easy Questions

1. Please evaluate the function $f(x) = 3x^3 - 2x^2 + x - 5$ at $x = 2$.

2. For the polynomial $g(x) = -4x^4 + 5x^2 - x + 1$, determine its degree and its leading coefficient.

3. Write the polynomial $h(x) = 5 - 3x + 2x^2 - x^3$ in standard form (descending powers of x). Also state the constant term.

4. Consider the polynomial $p(x) = x^3$. State its end behaviour as $x \to \infty$ and $x \to -\infty$.

5. Factor out the greatest common factor from $q(x) = 6x^4 - 9x^3 + 3x^2$.

Intermediate Questions

6. Expand the polynomial $(x + 2)^3$.

7. Use the Remainder Theorem to find the remainder when $f(x) = 2x^3 - 3x^2 + 4x - 1$ is divided by $(x - 1)$.

8. Factor completely the polynomial $x^3 - 6x^2 + 11x - 6$.

9. Determine the maximum number of turning points of any polynomial of degree 4.

10. Find the zeros of $f(x) = x^3 + 2x^2 - x - 2$ by grouping.

11. Use synthetic division to divide $f(x) = 2x^3 + 3x^2 - 5x + 6$ by $(x + 2)$ and state the quotient and remainder.

12. Factor completely $f(x) = x^3 - 4x$.

13. Describe the end behaviour of $f(x) = 2x^5 - 3x^3 + x - 4$.

14. Expand fully the function $f(x) = (x - 1)(x + 2)(x - 3)$.

15. Find all real zeros of $f(x) = x^4 - 5x^2 + 4$.

16. Show that $x = 2$ is a zero of $f(x) = 4x^3 - 8x^2 + 3x - 6$ using the Factor Theorem.

17. Determine whether the function $f(x) = x^4 - 2x^3 - 8x^2 + 12x$ is even, odd, or neither. Justify your answer.

18. Determine the multiplicity of the zero $x = 1$ for the function $f(x) = (x-1)^3(x+2)^2$.

19. Factor by grouping the polynomial $f(x) = x^5 - x^4 - 4x^3 + 4x^2$.

20. Prove that $f(x) = x^3 - 3x^2 + 3x - 1$ can be written as $(x - 1)^3$.

Hard Questions

21. Construct a polynomial function $f(x)$ of degree 4 that has zeros at $x = -2$ (multiplicity 1), $x = 1$ (multiplicity 2) and $x = 3$ (multiplicity 1). Write $f(x)$ in its factored form and then expand it fully, assuming a leading coefficient of 1.

22. Find a polynomial $f(x)$ of minimum degree such that $f(0) = 2$, $f(1) = 0$, $f(-1) = 0$ and $f(2) = 6$. Express your answer in factored form.

23. Factor the polynomial $f(x) = x^6 - x^5 - 7x^4 + x^3 + 6x^2$ as far as possible.

24. Use the Rational Root Theorem to list all possible rational zeros of $f(x) = 2x^4 - 3x^3 - 11x^2 + 12x + 9$.

25. Use Descartes' Rule of Signs to determine the maximum number of positive real zeros of $f(x) = x^5 - x^4 + 2x^3 - 4x^2 + x - 5$.

26. Divide $f(x) = 3x^5 - 2x^4 + x^3 - 7x^2 + 4x - 8$ by $x^2 - x + 1$ using polynomial long division and state the quotient and remainder.

27. For $f(x) = x^3 - 9x$, find all its zeros and their multiplicities. Then, sketch a rough graph of the function on pen and paper.

28. Analyse the effect of repeated zeros on the graph of $f(x) = (x - 2)^2(x + 1)^3$. Describe the behaviour of the graph near $x = 2$ and $x = -1$.

29. For the polynomial function $f(x) = x^7 - 2x^6 - x^5 + 4x^4 + x^3 - 3x^2 + 2x - 4$, determine the maximum number of turning points and describe its end behaviour.

30. Find a polynomial function of least degree with a leading coefficient of 2 that has zeros -3, 1 (with multiplicity 2) and 4. Express your final answer in expanded form.