ILLAWARRA TUTORING

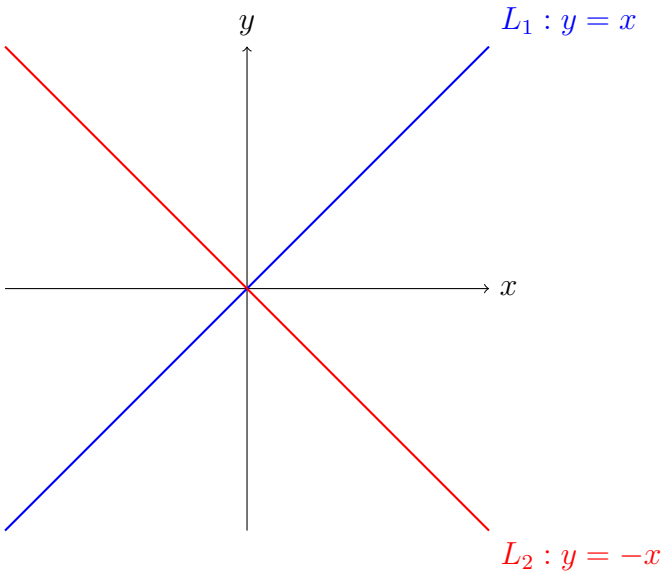
This worksheet will help you understand the conditions that make lines parallel or perpendicular and learn how to find their equations. You will practise determining slopes, writing equations for parallel and perpendicular lines, and solving more involved problems using these concepts.

Easy Questions

1. Find the slope of the line given by $y = 3x + 7$.
2. Determine whether the lines $y = 3x + 2$ and $y = 3x - 5$ are parallel.
3. Determine the slope of a line that is perpendicular to $y = 3x + 1$.
4. State the condition on the slopes of two lines for them to be parallel.
5. State the condition on the slopes of two lines for them to be perpendicular.

Intermediate Questions

6. Find the equation of the line that is parallel to $y = 4x - 3$ and passes through the point $(2, 1)$.
7. Find the equation of the line that is perpendicular to $y = -2x + 5$ and passes through the point $(1, -3)$.
8. Write the equation of the line that is perpendicular to the line joining $(0, 0)$ and $(4, 8)$ and passes through the point $(2, 3)$.
9. Using the diagram below, confirm that the lines L_1 and L_2 are perpendicular by calculating their slopes.



10. Find the x-intercept of the line that is parallel to $y = \frac{1}{2}x + 3$ and passes through $(4, -1)$.

11. Explain, using the example of the equations $y = mx + c$ and $y = mx + d$, why parallel lines must have equal slopes.

12. If a line is given by $y = 5x - 7$, what is the slope of any line perpendicular to it? Provide a justification.

13. Sketch a graph (using pen and paper) of $y = 3x + 2$ and the line perpendicular to it that passes through $(0, -1)$. Clearly label both lines and the point of intersection.

14. Find the equation of the line that is parallel to the line through $(1, 2)$ and $(3, 6)$ and passes through the origin.

15. Find the coordinates of the point where the line perpendicular to $y = -x + 4$ that passes through $(2, 3)$ intersects the line $y = -x + 4$.

16. Find the equation of the line perpendicular to $2x - 3y = 6$ and passing through the point $(3, 2)$.

17. Show algebraically that the lines $y = -\frac{1}{4}x + 7$ and $4y = -x + 28$ are parallel.

18. For the line $3x + 4y = 12$, find the equation of the line that is perpendicular to it and passes through $(4, -3)$.

19. Find the equation of the line that is parallel to $y = \frac{2}{3}x + 1$ but has a y-intercept of -2.

20. Determine the slope and y-intercept of the line that is perpendicular to $y = 4x + 9$.

Hard Questions

21. Given $y = 2x + 3$ and $y = -x + 4$, find the intersection point of these two lines and then determine the equation of the line that is perpendicular to $y = 2x + 3$ and passes through that intersection point.

22. Suppose a line has equation $y = mx + c$. A line perpendicular to it passes through $(5, -2)$ and is parallel to $y = -2x + 4$. Determine the values of m and c, and justify your answer.

23. Prove algebraically that two distinct lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$. Use examples to support your argument.

24. A line L has equation $y = ax + b$ and is perpendicular to $y = -\frac{1}{3}x + 7$. If L passes through $(6, 1)$, find a and b.

25. Determine the coordinates of the intersection of the following two lines: one is parallel to $y = x + 2$ with a y-intercept of 4, and the other is perpendicular to $y = x + 2$ and passes through $(0, -1)$.

26. Given $L_1 : y = 3x - 2$ and $L_2 : 2y = 6x + 4$, determine whether these lines are parallel or perpendicular. Show all your working.

27. A line with equation $y = 2x + b$ is reflected over the line $y = x$. If the original line has a slope of 2, what is the slope of its reflection? Provide justification.

28. Show that for any two parallel lines of the form $y = mx + c_1$ and $y = mx + c_2$, the vertical distance between them is constant. Provide a detailed explanation.

29. Find the equation of the line that is perpendicular to the segment joining $(1, -2)$ and $(5, 6)$ and that bisects this segment.

30. A triangle is formed by three lines:

$$L_1 : y = 2x + 1, \quad L_2 : y = -\frac{1}{2}x + 4, \quad \text{and} \quad L_3,$$

where L_3 is perpendicular to L_2 and passes through $(5, 0)$. Find the coordinates of the vertex of the triangle at the intersection of L_2 and L_3.