ILLAWARRA TUTORING

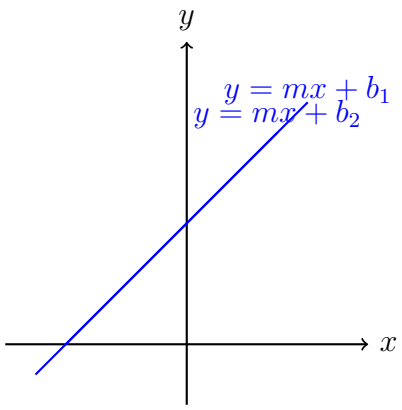
In this worksheet you will understand the conditions that make lines parallel or perpendicular and learn how to find their equations. You will practise identifying slopes, writing equations of lines given certain criteria, and justifying your results using algebraic reasoning.

Easy Questions

1. Write the slope of the line given by $y = 3x + 2$.
2. Find the equation of the line with slope 4 and y-intercept -1.
3. Are the lines $y = 2x + 5$ and $y = 2x - 3$ parallel?
4. Are the lines $y = \frac{1}{3}x + 2$ and $y = -3x + 1$ perpendicular?
5. If a line is perpendicular to $y = 4x + 7$, what is its slope?

Intermediate Questions

6. Find the slope of the line passing through $(1, 2)$ and $(5, 10)$. Is this line parallel to $y = 2x - 3$?
7. Find the value of k if the line $y = kx + 4$ is parallel to the line $y = 2x - 3$.
8. Find the value of k if the line $y = kx + 1$ is perpendicular to $y = -\frac{1}{2}x + 7$.
9. Below is a diagram showing two parallel lines. Determine the slope they share.



10. Convert $3x - 4y + 12 = 0$ into the form $y = mx + c$ and state the value of m.

11. Find the equation of the line parallel to $y = 5x - 3$ that passes through $(2, -1)$.

12. Find the equation of the line perpendicular to $y = -3x + 6$ that passes through $(-2, 4)$.

13. The line L has equation $y = \frac{1}{2}x + 2$. Find the value of b such that the line $y = \frac{1}{2}x + b$ passes through $(4, 4)$.

14. State the condition that two lines with slopes m_1 and m_2 must satisfy to be parallel.

15. State the condition that two lines with slopes m_1 and m_2 must satisfy in order to be perpendicular.

16. Given the line $y = \frac{3}{4}x - 2$ and the point $(0, 0)$, find the equation of the line perpendicular to it.

17. Find the y-intercept of the line parallel to $2x + 3y - 6 = 0$ that passes through $(-3, 2)$.

18. A line is perpendicular to $4x - y + 3 = 0$ and passes through $(1, -2)$. Find its equation.

19. For which value of m are the lines $y = mx + 1$ and $y = mx - 4$ distinct and parallel?

20. The instruction is: Write a brief explanation comparing the conditions that define parallel and perpendicular lines in terms of their slopes. Explain how the relationships between slopes differ for parallel and perpendicular lines.

Hard Questions

21. Prove that if two non-vertical lines are parallel then their slopes are equal.

22. Prove that if two non-vertical lines are perpendicular then the product of their slopes is -1.

23. Given the lines $L_1 : ax + by + c = 0$ and $L_2 : dx + ey + f = 0$, show that a necessary condition for L_1 and L_2 to be perpendicular is that $\dfrac{a}{b} \cdot \dfrac{d}{e} = -1$, assuming b and e are non-zero.

24. Find the condition on m such that the line $y = mx + c$ is perpendicular to $3x + 4y - 5 = 0$.

25. Find the equation of the line that is perpendicular to $y = 2x + 1$ and whose y-intercept is the same as that of the line parallel to $3x - y + 10 = 0$. Provide a justification for your answer.

26. Show that the lines $y = k(x - 1) + 2$ and $y = (2 - k)x + 2k + 1$ are perpendicular if and only if k satisfies a quadratic equation. Determine the possible values of k.

27. Show algebraically why two distinct parallel lines, each in the form $y = mx + c$ with the same m but different c, never intersect.

28. Given the line $y = mx + c$, prove that any line perpendicular to it can be written in the form $y = -\frac{1}{m}x + b$, and explain how b is determined if the line must pass through a specific point.

29. Verify that the lines $y = 7x + 3$ and $y = -\frac{1}{7}x - 8$ are perpendicular.

30. Consider the lines $L_1 : y = 3t\,x + t^2$ and $L_2 : y = -\frac{1}{3t}\,x + 2t - 1$, where $t \neq 0$. Find the value of t for which L_1 and L_2 are perpendicular, and justify your answer.