ILLAWARRA TUTORING

In this worksheet you will learn how to find the inverse of a function and understand the special relationship between a function and its inverse. You will practise finding inverses of various types of functions, verifying your answers by composition, and analysing the graph reflections about the line $y = x$.

Easy Questions

1. Given $f(x) = 2x + 3$, determine $f^{-1}(x)$.
2. Given $f(x) = x - 5$, determine $f^{-1}(x)$.
3. Given $f(x) = -x + 1$, determine $f^{-1}(x)$.
4. Evaluate using the inverse: If $f(x) = x + 7$, find $f^{-1}(10)$.
5. Explain in a short paragraph the relationship between a function and its inverse, particularly why their graphs are reflections about the line $y = x$.

Intermediate Questions

6. Given $f(x) = \dfrac{x-1}{4}$, determine $f^{-1}(x)$ showing all working.
7. Given $f(x) = 3x - 2$, determine $f^{-1}(x)$ and state the domain and range of both f and f^{-1}.
8. Given $f(x) = 4 - x$, show your working to determine $f^{-1}(x)$.
9. For $f(x) = 5x + 9$, first determine $f^{-1}(x)$ then show that $f(f^{-1}(x)) = x$.
10. Given $f(x) = 2x + 1$, determine $f^{-1}(x)$ and verify your answer by showing that $f(f^{-1}(x)) = x$.
11. Given $f(x) = x^2$ for $x \geq 0$, determine $f^{-1}(x)$ and state the domain and range of f^{-1}.
12. Draw and label the graphs of an invertible function and its inverse, including the line $y = x$.
13. Given $f(x) = \dfrac{2x+5}{3}$, determine $f^{-1}(x)$ showing your working.
14. Find and verify the inverse with instruction: Given $f(x) = 7x - 4$, determine $f^{-1}(x)$ and verify the result by composing f and f^{-1}.

15. Given $f(x) = \frac{1}{2}x + 8$, determine $f^{-1}(x)$ and state the domain and range.

16. Given $f(x) = \frac{x}{3} - 7$, determine $f^{-1}(x)$.

17. Given $f(x) = -3x + 2$, prove that $f^{-1}(x) = \frac{2 - x}{3}$ and verify your answer by composing f and f^{-1}.

18. Given $f(x) = 4 - 2x$, determine $f^{-1}(x)$ showing all algebraic steps.

19. Given $f(x) = \frac{3}{x + 1}$, determine $f^{-1}(x)$. Clearly state any restrictions on the domain for which the inverse exists.

20. For a linear function $f(x) = ax + b$, find the values of a and b for which f is self-inverse (i.e. $f(x) = f^{-1}(x)$). Show all working.

Hard Questions

21. For $f(x) = \frac{2 - x}{3}$, first determine $f^{-1}(x)$ and then find all values of x for which $f(x) = f^{-1}(x)$. Provide detailed working.

22. Given $f(x) = \frac{x + 2}{2x - 3}$, determine $f^{-1}(x)$. Also, state any restrictions on the domain of f where it is invertible.

23. For the function $f(x) = \frac{3x - 4}{x + 5}$ (with $x \neq -5$), find $f^{-1}(x)$ and verify your answer by showing that $f(f^{-1}(x)) = x$.

24. Prove that if a function f is invertible then its inverse f^{-1} is also invertible and that $\left(f^{-1}\right)^{-1} = f$. Provide a formal explanation.

25. Given $f(x) = \sqrt{x + 3}$ for $x \geq -3$, determine $f^{-1}(x)$ and state the domain of f^{-1}.

26. For $f(x) = x^3 + 1$, determine $f^{-1}(x)$ and show all steps.

27. Given $f(x) = \frac{2x + 3}{x - 1}$ (with $x \neq 1$), determine $f^{-1}(x)$ and clearly state any domain restrictions.

28. Explain why the function $f(x) = |x|$ is not invertible over the real numbers. Then, state a restriction on the domain of f that makes it invertible and find the inverse function for that restricted domain.

29. Consider the function $f(x) = e^x$. Find the inverse function $f^{-1}(x)$ and state its domain. (Recall that the inverse of the exponential function is the natural logarithm.)

30. For the linear function $f(x) = 3x + 1$, first determine $f^{-1}(x)$. Then, explain briefly why the graph of f and the graph of $f^{-1}(x)$ are reflections of each other about the line $y = x$.