ILLAWARRA TUTORING

This worksheet focuses on inverse functions. You will learn how to find the inverse of a function algebraically and understand the special relationship between a function and its inverse. Recall that to find the inverse of a function you swap the roles of the variable and the function output and then solve for the new output variable.

Easy Questions

1. Find the inverse of the function $f(x) = 2x + 3$.
2. Find the inverse of the function $f(x) = x - 7$.
3. State in one or two sentences the reflection property shared by a function and its inverse.
4. Find the inverse of the function $f(x) = -x$.
5. Find the inverse of the function $f(x) = 3x$.

Intermediate Questions

6. Find the inverse of the function $f(x) = \dfrac{x - 5}{4}$.
7. Find the inverse of the function $f(x) = 4x + 1$.
8. Find the inverse of the function $f(x) = 2 - x$.
9. Find the inverse of the function $f(x) = -3x + 7$.
10. Find the inverse of the function $f(x) = \dfrac{1}{2}x - 4$.
11. Explain why a function must be one-to-one in order for its inverse to also be a function.
12. Find the inverse of the function $f(x) = \dfrac{x + 2}{x - 1}$. In your answer state any restrictions on the domain.
13. Find the inverse of the function $f(x) = \dfrac{2x - 3}{5}$.
14. For the function $f(x) = 3x + 1$ with inverse $f^{-1}(x) = \dfrac{x - 1}{3}$, verify by composing f and f^{-1} that $f(f^{-1}(x)) = x$.

15. Find the inverse of the function $f(x) = \dfrac{1}{x+2}$ and state the domain of the inverse.

16. Find the inverse of the function $f(x) = -\dfrac{x}{3} + 2$.

17. Find the inverse of the function $f(x) = \dfrac{5-2x}{3}$.

18. Find the inverse of the function $f(x) = \dfrac{4}{x-1}$.

19. Find the inverse of the function $f(x) = \dfrac{x+1}{2x+3}$.

20. Using pen and paper, draw a diagram that shows a function and its inverse as reflections across the line $y = x$. Label both the function and its inverse.

Hard Questions

21. Suppose $f(x) = \dfrac{ax+b}{cx+d}$ with $ad-bc \neq 0$. Derive a general expression for the inverse function $f^{-1}(x)$.

22. Let $f(x) = \dfrac{2x-1}{3x+4}$. Prove by algebraic manipulation that the inverse function is

$$f^{-1}(x) = \frac{-4x-1}{3x-2}\,.$$

23. Consider the function $f(x) = x + \dfrac{1}{x}$ for $x \neq 0$. Determine whether f has an inverse function over its entire domain. Justify your answer.

24. The function $f(x) = \sqrt{x}$ has inverse $f^{-1}(x) = x^2$ when its domain is $x \geq 0$. Explain why it is necessary to restrict the domain of $g(x) = x^2$ in order for its inverse to be a function.

25. Prove that if a function f is invertible then its inverse function f^{-1} is also invertible and that $(f^{-1})^{-1} = f$. Provide a complete proof using the definition of inverse functions.