ILLAWARRA TUTORING

In this worksheet you will learn how to find the inverse of a function and understand the special relationship between a function and its inverse. You will practise finding inverses, verifying the symmetry about the line $y = x$, and exploring the properties of inverse functions.

Easy Questions

1. Find the inverse of the function. Write your answer in the form $f^{-1}(x)$.
   $f(x) = 2x + 5$
2. Determine $f^{-1}(x)$ for the function.
   $f(x) = x - 3$
3. Decide if the function is its own inverse. Explain your reasoning.
   $f(x) = \frac{1}{x}$
4. Find the inverse of the function.
   $f(x) = -x$
5. Find $f^{-1}(x)$ when
   $f(x) = \frac{x+1}{2}$

Intermediate Questions

6. Compute the inverse of the function.
   $f(x) = 3x - 4$
7. Find $f^{-1}(x)$ for the function.
   $f(x) = \frac{1}{2}x + 7$
8. Determine the inverse of the function.
   $f(x) = -2x + 6$
9. Find the inverse of the function.
   $f(x) = \frac{x-2}{3}$
10. Determine $f^{-1}(x)$ for the function.
    $f(x) = 5 - x$

11. Calculate the inverse of the function given by
$f(x) = 4x + 1$

12. For the function $f(x) = \frac{x}{3}$, verify that its graph and the graph of $f^{-1}(x)$ are reflections about the line $y = x$. Explain your reasoning.

13. Show that for $f(x) = 2x - 5$, composing f with f^{-1} returns the original input. That is, prove $f(f^{-1}(x)) = x$.

14. Find the inverse of the function.
$f(x) = 1 - x$

15. Determine $f^{-1}(x)$ for the function.
$f(x) = \frac{3x + 4}{2}$

16. Compute the inverse of the function.
$f(x) = \frac{2x - 3}{5}$

17. Find the inverse of the function given by
$f(x) = \frac{7 - 2x}{3}$

18. Determine $f^{-1}(x)$ when
$f(x) = -3x + 2$

19. Find the inverse function $f^{-1}(x)$ for
$f(x) = \frac{x + 3}{4}$

20. For the function $f(x) = \frac{1}{x - 2}$, find $f^{-1}(x)$. In your answer also state clearly the domain of f and f^{-1}.

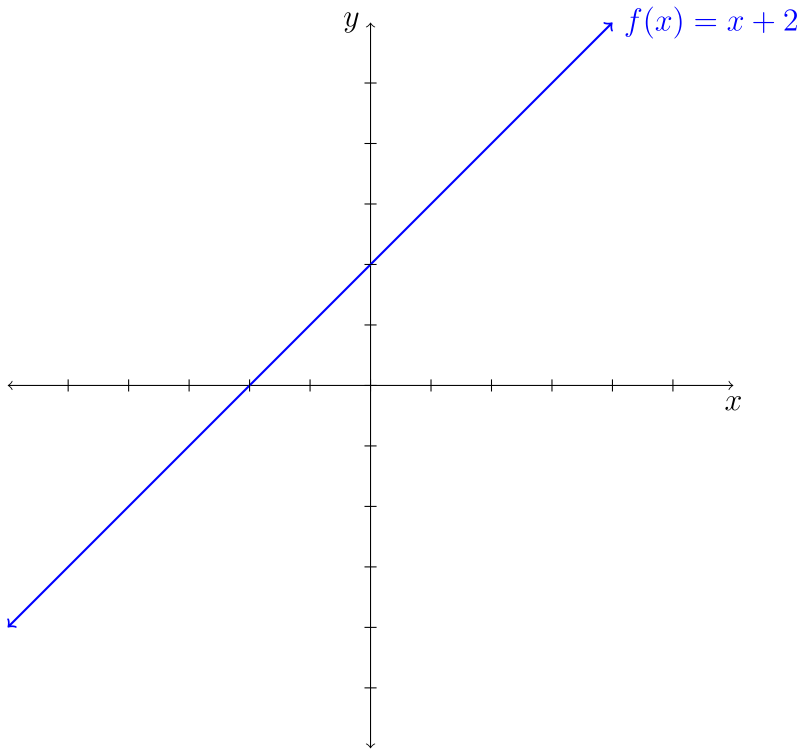
Hard Questions

21. Prove that the graph of a function and its inverse are symmetric about the line $y = x$. Use the function $f(x) = 2x + 3$ as an example to illustrate your proof.

22. For $f(x) = \frac{3x - 1}{4}$, first derive $f^{-1}(x)$. Then, compute the composition $f(f^{-1}(x))$ and verify that it simplifies to x.

23. Let $f(x) = \frac{-x + 5}{2}$. Find $f^{-1}(x)$ and show by direct calculation that $f(f^{-1}(x)) = x$.

24. The graph of $f(x) = x + 2$ is shown in the diagram below. Using the diagram as a reference, sketch the graph of $f^{-1}(x)$ on the same set of axes.



Write a short explanation of how the graph of $f^{-1}(x)$ relates to the graph of $f(x)$.

25. Consider the function $f(x) = \dfrac{x+2}{x-1}$. Discuss the conditions needed for f to be invertible. Then, find $f^{-1}(x)$ for the valid domain.

26. For the function $f(x) = \sqrt{x+3}$, determine the inverse function $f^{-1}(x)$. Include a discussion of the necessary domain restrictions for both f and f^{-1}.

27. Find the inverse of $f(x) = \dfrac{1}{3x+2}$. Clearly state any restrictions on the variable that are required for the function and its inverse.

28. Prove that if f is invertible then $\left(f^{-1}\right)^{-1} = f$. Illustrate your proof by finding the inverse of $f^{-1}(x)$ when $f(x) = 5x - 4$.

29. Let f be an invertible function. Explain why $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$ for all x in the appropriate domains. Then, provide an example using $f(x) = 2x + 1$ to support your explanation.

30. For the function $f(x) = \dfrac{2x+3}{x-4}$, determine the inverse function $f^{-1}(x)$. Then, verify your answer by demonstrating that $f^{-1}(f(x)) = x$.