ILLAWARRA TUTORING

In this worksheet you will develop the skills to find the intersection points of graphs both algebraically and visually. Work carefully through each question and show all steps in your solutions.

Easy Questions

1. Find the intersection point of the graphs of

$$f(x) = x + 1$$

and

$$g(x) = -x + 3.$$

Write down the solution of the equation created by equating the two functions.

2. Determine the intersection of the graphs of

$$f(x) = 2x + 1$$

and

$$g(x) = 2x - 4.$$

State whether an intersection exists and justify your answer.

3. Identify the intersection point of the graphs of

$$f(x) = x - 2$$

and

$$g(x) = -x + 4.$$

4. Explain in your own words what is meant by "the intersection of two graphs". Provide an example to support your explanation.

5. Describe the different possibilities for the number of intersection points between two graphs. In your answer include an example of two functions that intersect in exactly one point, two functions with no intersection, and two functions that have infinitely many intersection points.

Intermediate Questions

6. Find algebraically the intersection points of

$$f(x) = x^2$$

and

$$g(x) = x + 2.$$

Show all steps leading to your solution.

7. Determine the intersection points of

$$f(x) = x^2 - 4$$

and

$$g(x) = x.$$

Express your answer in exact form.

8. Find the points of intersection between

$$f(x) = 2x^2 + x - 1$$

and

$$g(x) = x + 1.$$

Solve the resulting equation and state the coordinates of the intersections.

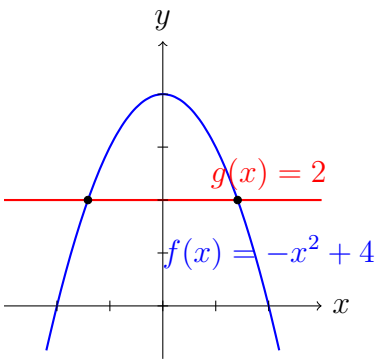
9. Using the diagram below, identify the intersection points between

$$f(x) = -x^2 + 4$$

and

$$g(x) = 2.$$

Then, solve the equation $-x^2 + 4 = 2$ to verify your answer.



10. Verify the solution for the intersection of

$$f(x) = x^2 - 1$$

and

$$g(x) = 2x - 1.$$

Solve the algebraic equation and check your answer.

11. Determine the intersection points of

$$f(x) = x^2$$

and

$$g(x) = 3x + 1.$$

Express your answers in surd form where necessary.

12. Find the coordinates of the intersection of

$$f(x) = -x^2 + 3x + 2$$

and

$$g(x) = x + 2.$$

Explain your algebraic steps.

13. Determine the intersection points of

$$f(x) = x^2 + 2x$$

and

$$g(x) = 2x + 2.$$

Solve for x and then find the corresponding y values.

14. Find the intersection of

$$f(x) = x^2 - x - 2$$

and

$$g(x) = x - 4.$$

Solve the equation and state why there are no real solutions.

15. Solve for the intersection of

$$f(x) = x^2 + 4$$

and

$$g(x) = 3x + 1.$$

Show your working and explain why no real intersection points exist.

16. Determine the intersection points of

$$f(x) = -2x^2 + x + 3$$

and

$$g(x) = x + 1.$$

Solve the equation and provide your answers in exact form.

17. Find the intersection points of

$$f(x) = x^2 + x - 2$$

and

$$g(x) = -x^2 + 5.$$

Show all steps in your solution and express your answers in surd form if necessary.

18. Determine the intersection of

$$f(x) = 3x^2 - x + 1$$

and

$$g(x) = 2x^2 + 2x - 3.$$

Solve the resulting equation and discuss why there are no real intersection points.

19. Find the points of intersection of

$$f(x) = x^2 - 4x + 3$$

and

$$g(x) = -x^2 + 6x - 5.$$

Factorise and solve the equation, then state the coordinates of the intersection points.

20. Calculate the intersection of

$$f(x) = 2x^2 + x - 1$$

and

$$g(x) = -x^2 + 4x + 3.$$

Solve your equation and express the answers in exact form.

Hard Questions

21. Find all the intersection points of

$$f(x) = x^3 - 2x^2 - x + 2$$

and

$$g(x) = x^2 - 1.$$

Hint: Rearrange the equation and look for common factors.

22. Determine the intersection points of

$$f(x) = 2x^2 + 3x - 5$$

and

$$g(x) = x.$$

Express your solutions in exact form.

23. Find the intersection points of

$$f(x) = -x^2 + 4$$

and

$$g(x) = 3x - 2.$$

Solve the equation and provide the answers in surd form.

24. Determine the intersection points of

$$f(x) = x^3$$

and

$$g(x) = x - 2.$$

Since the cubic equation $x^3 - x + 2 = 0$ does not factorise easily, use an appropriate method to estimate the real solution to two decimal places.

25. Find the intersection of

$$f(x) = x^2 + 2x + 1$$

and

$$g(x) = -x^2 + 6x - 8.$$

Solve the equation and explain why there are no real intersection points.

26. Find all the intersection points of

$$f(x) = 3x^3 - x$$

and

$$g(x) = x^2 + 2.$$

Set up the equation, and if necessary, use a graphing method to approximate the real solution(s). Explain your method.

27. Two projectiles are launched and their heights (in metres) at time t (in seconds) are given by

$$h_1(t) = -5t^2 + 20t + 2$$

and

$$h_2(t) = -5t^2 + 15t + 7.$$

Find the time at which the projectiles are at the same height. Show all your working.

28. Determine the intersection points of

$$f(x) = x^3 - 3x$$

and

$$g(x) = 2x + 1.$$

Solve the equation $x^3 - 3x = 2x + 1$ and, if necessary, use a graphical or numerical method to estimate the real solution(s).

29. Sketch by hand the graphs of

$$f(x) = x^2$$

and

$$g(x) = 4.$$

Then, mark the intersection points on your sketch and justify your answer algebraically.

30. Find all the intersection points of

$$f(x) = x^3 - 4x$$

and

$$g(x) = 2x.$$

Solve the equation $x^3 - 4x = 2x$ and express your answers in exact form.