In this worksheet you will learn to recognise and determine whether a given relation is a function. Recall that a function is a relation in which each input is paired with a unique output. Work carefully through the questions and justify your answers.

Easy Questions

1. In your own words, define what a function is.
   Answer Hint: A function is a relation where every input is paired with exactly one output, ensuring no input is assigned two different outputs.

2. Decide whether the relation represented by the set of ordered pairs $(1, 2)$, $(3, 4)$, $(5, 6)$ is a function. Justify your answer.

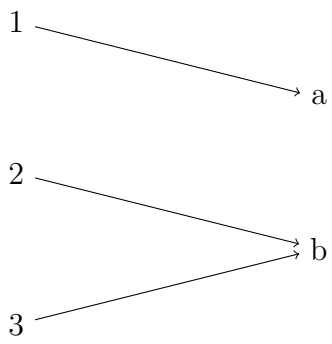
3. Determine if the relation defined by the set $(2, 3)$, $(4, 5)$, $(2, 7)$ is a function. Explain why or why not.

4. Provide an example of a relation (expressed as a set of ordered pairs) that does not define a function. Explain your reasoning.

5. State whether the following statement is True or False and explain: "A relation that contains both (a, b) and (a, c), where $b \neq c$, can be a function."

Intermediate Questions

6. Consider the mapping diagram below. Decide whether it represents a function. Explain your reasoning.



7. Explain in a short paragraph why the vertical line test is used to determine whether a graph represents a function.

8. Consider the following table:

x	y
1	3
2	5
3	7
4	9

   Decide whether this relation is a function and explain why.

9. Given the set of ordered pairs $(0, 0)$, $(1, 2)$, $(2, 4)$, $(3, 6)$, confirm that this relation is a function and justify your answer.

10. A relation is a function if and only if every input is paired with a _____ output.

11. Explain why it is essential for a relation to assign *only one* output to each input in order to be a function.

12. Examine the table below and state whether the relation is a function. Justify your answer.

x	y
2	4
2	5
3	6

13. Consider a relation where the input 7 is paired with 10 and also with 12. Explain why this violates the definition of a function.

14. Construct a relation on the set $1, 2, 3$ (expressed by listing the ordered pairs) that is a function.

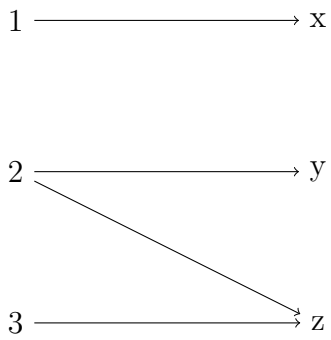
15. Consider the following rule: "For any integer, if the input is even then assign the output as twice the input; if the input is odd, assign the output as the input." Determine whether this rule defines a function. Explain your reasoning.

16. Let $A = 1, 2, 3$ and $B = a, b$. Explain how many relations from A to B are functions. (Hint: For each input there are a fixed number of choices.)

17. Define what is meant by an ordered pair and explain why ordered pairs are fundamental in the description of a function.

18. Given the sets $X = red, blue$ and $Y = circle, square$, list all possible relations from X to Y that are functions. How many such functions are there? Justify your answer.

19. Look at the following diagram and decide if it represents a function. Draw your conclusion and explain.



20. Provide a brief explanation of why a function cannot assign more than one output to a single input.

Hard Questions

21. Prove that if A is a set with n elements and B is a set with m elements, then the number of functions from A to B is m^n. Provide a clear explanation of each step.

22. Discuss the conditions under which a relation defined as a set of ordered pairs qualifies as a function. Your explanation should include the concept of uniqueness of outputs.

23. Define a relation on the set of integers as follows: For every integer x, if $x \geq 0$ assign x^2 and if $x < 0$ assign $|x|$. Determine whether this relation is a function. Justify your answer.

24. Explain in detail the significance of the uniqueness condition in the definition of a function. Provide at least two examples to support your explanation.

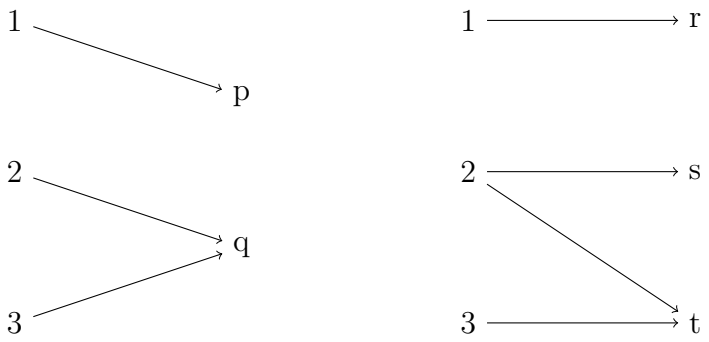
25. Construct a relation on the set a, b, c, d (by listing ordered pairs) that is not a function. Explain precisely why your relation fails to be a function.

26. Taking the relation you created in the previous question, modify it so that it becomes a function. Explain what changes were made and why they ensure the relation becomes a function.

27. Design a relation that pairs the names of five students (you may choose any names) with their favourite colours. Explain the criteria that ensure your relation qualifies as a function.

28. Suppose a relation is defined by a set of ordered pairs but not every possible element of a predetermined set appears as an input. Discuss if this absence of some inputs violates the definition of a function. Explain your reasoning.

29. Examine the two diagrams below and state which one(s) represent a function. Justify your answers.



30. In your own words, summarise the essence of a function. Be sure to include a discussion of why each input must be associated with a unique output.