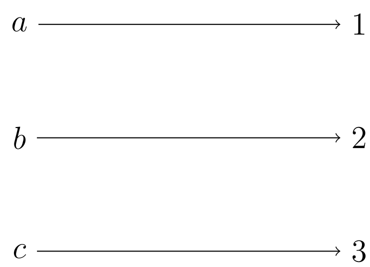
ILLAWARRA TUTORING

In this worksheet you will learn to understand a function as a relation where each input is paired with a unique output. You will explore examples and non-examples, using ordered pairs and mapping diagrams to verify whether a relation qualifies as a function.

Easy Questions

1. Write in your own words what it means for a relation to be a function. In your answer, explain why each input must have exactly one output.
2. Consider the relation given by $(1, 2), (2, 4), (3, 6)$. Write a brief explanation stating whether this relation is a function and why.
3. A rule is described verbally as "each number is doubled to produce its output". If the input is 5, what is the unique output? Briefly justify why this rule defines a function.
4. Below is a mapping diagram. Determine whether the relation is a function. Write your answer with a short explanation.



5. Provide a real-life example of a function (a relation with one output per input) and explain briefly why your example qualifies as a function.

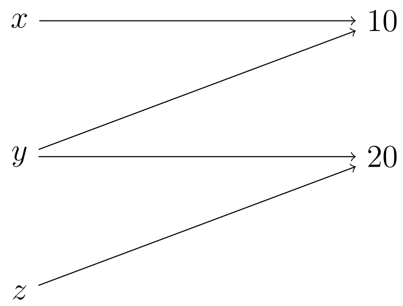
Intermediate Questions

6. Consider the relation $(2, 5), (3, 7), (4, 9), (5, 11)$. List the set of all inputs and the set of all outputs. Is this relation a function? Explain your reasoning.
7. A relation is represented by the following table:

Input	Output
1	3
2	5
3	7
4	9

State whether or not this relation is a function and provide a short explanation.

8. Examine the following mapping diagram. Does it represent a function? Justify your answer.



9. Explain why the condition that "each input has exactly one output" is essential in the definition of a function. Provide one example where this condition fails.

10. You are given two relations:

$$R_1 = (1,2),(2,3),(3,4) \quad \text{and} \quad R_2 = (1,2),(1,3),(2,4).$$

Identify which relation is a function and explain your choice.

11. The relation is given by the rule: "each input is increased by 3". For the domain $1, 2, 3, 4$, write the relation as a set of ordered pairs and explain why it qualifies as a function.

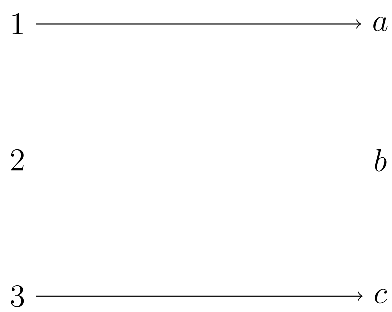
12. Consider the relation $S = (2,4),(2,5),(3,6)$. Explain why this relation is not a function. Then, remove the minimum number of ordered pairs from S to obtain a function and list the new relation.

13. A relation is defined on the set a, b, c, d and is represented by $(a,1),(b,2),(d,4)$. List the domain and codomain as given and discuss whether any input lacks an output. Is the relation a function?

14. A relation on the set of letters p, q, r is defined as follows: "each letter is assigned its position in the alphabet". Write this relation in set notation for the given domain and confirm that this relation satisfies the condition to be a function.

15. Explain why it is important to specify the domain of a relation when determining whether it is a function. Provide an example illustrating your explanation.

16. A mapping diagram is partially given as follows:



Add an arrow for input 2 such that the complete mapping represents a function. Write down the new complete set of ordered pairs.

17. Consider the relation represented by the set $(0,0),(1,2),(2,4),(3,6)$. Provide a short justification for why this relation meets the definition of a function.

18. Two relations are given as follows:

$$A = (x,y) \mid x \in 1,2,3,\ y = x+1 \quad \text{and} \quad B = (1,2),(2,3),(1,4).$$

Without using function notation, determine which one is a function and explain why.

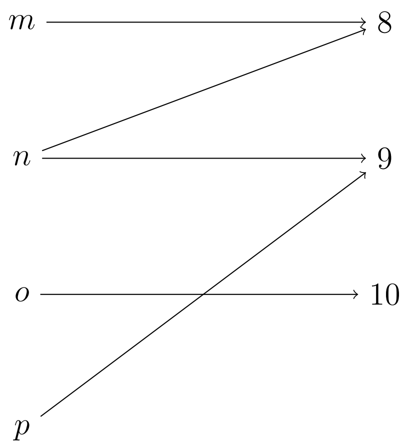
19. Draw (using pen and paper) a diagram that represents the relation $(a,3),(b,5),(c,7)$. Be sure to clearly indicate the domain and codomain, and write a sentence explaining how your diagram shows that the relation is a function.

20. A student claims that the following set $(1,2),(2,3),(2,4),(3,5)$ represents a function. Identify the error in this claim and explain how it violates the definition of a function.

Hard Questions

21. Let the relation be defined by $R = (x,y) \mid x \in 1,2,3,4,\ y = 2x-1$. Use set-builder notation to express this relation and explain in detail why it defines a function.

22. Analyse the following mapping diagram. Determine if the relation is a function and write a detailed explanation.



23. Provide an example of a relation (expressed as a set of ordered pairs) that is not a function. Explain clearly what property is violated.

24. A relation is described by the following: "Each even number from $2,4,6,8$ is paired with half its value." Write this relation as a set of ordered pairs and explain why it represents a function.

25. Consider the relation defined on $1, 2, 3, 4, 5$ by the rule: "If the input is less than or equal to 3, the output is the input increased by 2, otherwise the output is the input decreased by 1." Write the set of ordered pairs for this relation and confirm that it is a function with a detailed explanation.

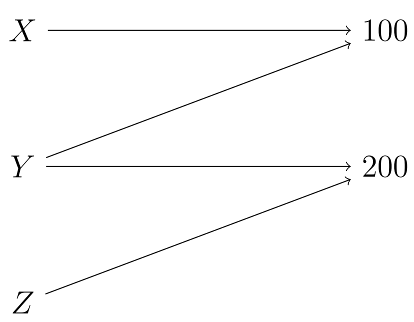
26. The same relation is given in two ways:

$$C = (a, a^2) \mid a \in 1, 2, 3 \quad \text{and} \quad D = (1, 1), (2, 4), (3, 9), (3, 8).$$

Determine which of the representations is a function and justify your answer.

27. Given the relation $T = (5, 10), (6, 12), (7, 14), (8, 16)$, list the domain and the range. Then, explain why these lists confirm that T is a function.

28. A mapping diagram is drawn as follows:



Identify the issue in this diagram that prevents the relation from being a function. Describe how you would modify the relation (by removing an arrow) so that it satisfies the definition of a function.

29. Describe a real-world situation where the relation between two quantities qualifies as a function. In your explanation, identify the domain and discuss how the unique output condition is satisfied.

30. Design two distinct relations on the set $1, 2, 3, 4$: one that is a function and one that is not. Write each relation as a set of ordered pairs and provide a clear explanation for why one satisfies the definition of a function and the other does not.