ILLAWARRA TUTORING

This worksheet will help you grasp the idea of a function as a relation where each input is paired with a unique output.

Easy Questions

1. Consider the relation $(1,4),(2,5),(3,6)$. Is this relation a function? Explain your answer.

2. Look at the relation $(1,2),(2,3),(1,5)$. Does every input have a unique output? State your reasoning.

3. Below is a mapping diagram. Decide if this relation is a function.

   $1 \longrightarrow 2$

   $2 \longrightarrow 4$

   $3 \longrightarrow 6$

4. Consider the table of values below:

x	y
0	1
1	3
2	5

   Confirm that this relation is a function.

5. Write your own example of a relation that is a function. Make sure your example shows that each input is paired with a unique output.

Intermediate Questions

6. Consider the rule "for every number x, let $y = x^2$". Is this relation a function? Provide your explanation.

7. Examine the relation $(-2,4),(-2,3),(0,0),(3,9)$. Determine whether this relation is a function and justify your answer.

8. A relation is represented by the following mapping diagram. Decide if it is a function.

$1 \longrightarrow 2$

$2 \longrightarrow 4$

$3 \longrightarrow 6$

9. Given the table:

x	y
1	2
2	4
3	6
4	8

explain why this relation qualifies as a function.

10. A teacher assigns each student their unique student number. Discuss why this rule defines a function.

11. Define a relation where for each integer x (from 1 to 10) the output y is the number of vowels in the English name of x. Does each input yield a unique output? Explain.

12. Consider the relation "assign to every person their age". Is this a function? Provide justification, noting that different persons may share the same age.

13. Can a function assign the same output to two different inputs? Provide an example to support your answer.

14. Consider the relation $(2, 5), (3, 5), (4, 5)$. Verify that this relation meets the definition of a function.

15. Write down, in your own words, the definition of a function.

16. Consider the rule: "For every positive integer x, if x is even let $y = 10$, and if x is odd let $y = 5$". Is this relation a function? Explain your reasoning.

17. Propose a rule that defines a function. Explain clearly how your rule ensures that each input is paired with exactly one output.

18. Explain why the rule "y is any number such that $y^2 = x$" does not define a function when the domain is the set of positive numbers.

19. Explain why having unique inputs in a table of ordered pairs guarantees that the relation is a function.

20. Provide an everyday example of a function and explain why it meets the criteria of having each input paired with a unique output.

Hard Questions

21. Consider the relation on the set of integers defined by: for each integer x, if x is prime let $y = 1$, otherwise let $y = 0$. Is this relation a function? Provide a detailed justification.

22. Examine the rule "assign to each book the year in which it was first published". Does this rule define a function? Justify your answer.

23. Given the relation defined on the set of real numbers by the rule "for each x, let y be any number satisfying $y^2 - x = 0$", explain why this relation does not define a function.

24. Construct a rule that defines a function on a non-empty set A and a set B. Your rule must ensure that at least two distinct elements of A are assigned the same element of B. Write down your rule and explain why it meets the definition of a function.

25. Discuss whether the statement "every relation from the set of real numbers to the set of real numbers is a function" is true or false. Provide justification for your answer.

26. Consider the relation defined on the set $2, 4, 6$ by assigning to each x its greatest proper divisor. Determine if this relation is a function and explain your reasoning.

27. Suppose a relation is defined by the set of ordered pairs $(a, b), (c, d), (a, e)$. Explain in detail why this relation does not qualify as a function, identifying the specific property that is violated.

28. Let a relation R be defined on the set of integers by: for each integer x, y is the remainder when x is divided by 3. Determine whether R is a function and justify your answer.

29. Prove that if a relation defined on a set A is a function, then for any element x in A the relation assigns at most one output. Provide a clear explanation.

30. Define what is meant by a constant function using a set of ordered pairs as an example. Explain why your example fulfils the criteria of a function.