ILLAWARRA TUTORING

In this worksheet you will explore cubic functions and their graphs, and learn how they differ from linear and quadratic functions. You will answer questions ranging from basic evaluations and transformations to more in-depth analysis of cubic behaviour.

Easy Questions

1. Evaluate the function $f(x) = x^3$ at $x = 2$. What is the value of $f(2)$?
2. Identify the degree and the leading coefficient of $f(x) = 2x^3$.
3. Sketch by hand the graph of $f(x) = x^3$. Label the points $(-1, -1)$, $(0, 0)$ and $(1, 1)$ on your graph.
4. Describe in words how the graph of $f(x) = x^3$ changes when it is transformed to $f(x) = -x^3$. What is the effect on the graph?
5. Find the x-intercept of the function $f(x) = x^3$. Justify your answer.

Intermediate Questions

6. Factorise $f(x) = x^3 - 3x + 2$ completely and list all its real roots.
7. Determine the y-intercept of the function $f(x) = 2x^3 - 5x^2 + x - 2$.
8. Show that the function $f(x) = -x^3 + 4x$ passes through the origin.
9. Compare the end behaviour of $f(x) = x^3$ and $g(x) = -2x^3$. Explain how the leading coefficient affects the graphs as $x \to \infty$ and $x \to -\infty$.
10. Describe the symmetry of the function $f(x) = x^3$. Is the function even, odd or neither? Explain your answer.
11. Without using calculus, determine the inflection point of $f(x) = x^3$.
12. Factorise $f(x) = x^3 - 6x^2 + 11x - 6$ completely and list the x-intercepts of the graph.
13. Draw a diagram of $f(x) = x^3 - x$ and label the points corresponding to $(-1, -1)$, $(0, 0)$ and $(1, 1)$.
14. Describe the transformation that changes $f(x) = x^3$ into $f(x) = x^3 + 2$. What happens to the graph?

15. Consider $g(x) = (x-1)^3$. Describe the horizontal shift relative to $f(x) = x^3$.
16. Identify the transformations applied to $f(x) = x^3$ to obtain $h(x) = -2(x+3)^3$. Explain each transformation.
17. Explain how many real roots a cubic function can have and under what circumstances repeated roots occur.
18. For a depressed cubic of the form $f(x) = x^3 + px + q$, describe generally how the coefficient p influences the shape of the graph.
19. Solve $f(x) = (x+2)^3 = 0$. State the multiplicity of the root.
20. Compare the rate of change of a quadratic function with that of a cubic function as $|x|$ becomes very large. Explain your answer.

Hard Questions

21. Explain in detail how in a cubic function $f(x) = ax^3 + bx^2 + cx + d$ the sign of the leading coefficient a determines the end behaviour of the graph as $x \to \infty$ and $x \to -\infty$.
22. Without using calculus, explain how one might estimate the turning points of the cubic function $f(x) = 3x^3 - 9x$. Include in your answer how symmetry or other features of the graph can assist in your estimation.
23. For the function $f(x) = x^3 - 3x^2 + k$, determine the value of k so that the graph passes through the point $(2, -2)$.
24. Given $f(x) = (x-2)^3 + 1$, find the x-coordinate for which $f(x) = 1$.
25. Factorise $f(x) = x^3 - 12x + 16$ completely and determine the nature (including multiplicity) of its real roots.
26. For a cubic function in the form $f(x) = x^3 + bx^2 + cx + d$, explain how modifying b affects the horizontal position of the function's point of inflection.
27. Consider the functions $f(x) = x^3$ and $g(x) = x^3 + 3x^2$. Explain how the addition of the term $3x^2$ affects the graph of the cubic function.
28. Explain the difference between the graphs of $f(x) = (x+1)^3$ and $g(x) = -(x+1)^3$. What effect does the negative sign have on the shape of the graph?
29. A cubic function f satisfies $f(-2) = 0$, $f(1) = 0$ and $f(3) = 16$. Express $f(x)$ in the form $f(x) = k(x+2)(x-1)(x-a)$ and determine the values of both k and a.
30. Explain why every cubic function has at least one real root, using the end behaviour of cubic functions as part of your explanation.