



This worksheet focuses on cubic functions and their graphs. You will explore the properties of cubic functions, compare them with linear and quadratic functions, and examine transformations and factorisations specific to cubics. All mathematical expressions are written inline with display style. Read each instruction carefully and answer all questions.

## Easy Questions

1. Write down the domain and range of the function  $f(x) = x^3$ .
2. Using pen and paper, draw a rough sketch of the graph of  $f(x) = x^3$ . Include the origin and indicate the end behaviour.
3. Consider  $f(x) = 2x^3 - 3x$ . Calculate  $f(-2)$ .
4. Given  $f(x) = x^3$ , write the expression for  $f(x - 1)$  and describe in words how the graph is shifted compared to  $f(x)$ .
5. In a short paragraph, explain one key difference between the graph of a cubic function and the graphs of linear and quadratic functions.

## Intermediate Questions

6. Factorise the function  $f(x) = x^3 - x$  completely and state its zeros.
7. Determine all zeros of the function  $f(x) = x^3 - 3x^2 + 2x$  by factorising it completely. Write your answer in ascending order.
8. Describe the end behaviour of the cubic function  $f(x) = -2x^3 + 5$ . Explain what happens as  $x \rightarrow \infty$  and as  $x \rightarrow -\infty$ .
9. Factorise the expression  $f(x) = x^3 + 1$  completely using an appropriate formula.
10. Investigate the symmetry of the function  $f(x) = -x^3$ . State whether the function is odd, even, or neither and justify your answer.
11. For the function  $f(x) = x^3 - 3x$ , compute  $f(2)$  and  $f(-2)$ . Write both results.
12. Find the equations of the reflections of  $f(x) = x^3$  in the x-axis and in the y-axis. Write a brief explanation for each transformation.
13. Given  $f(x) = 2x^3 - 4x$ , find the y-intercept and the x-intercepts of the function.

14. Consider the transformation leading to  $f(x) = (x - 1)^3 + 2$ . Explain how the graph of  $f(x) = x^3$  is shifted horizontally and vertically to obtain the new graph.
15. Using pen and paper, sketch the graph of  $f(x) = x^3 - 3x + 2$ . Ensure you mark the intercepts and note any turning points you can deduce from your sketch.
16. Compare the graphs of  $f(x) = x^3$  and  $g(x) = x^3 + 6$ . Describe how the graph is transformed from  $f(x)$  to  $g(x)$ .
17. Find a cubic function with leading coefficient 1 that has zeros at  $x = -2, 0$ , and  $3$ . Write the function in factorised form and expand it.
18. Factorise completely the expression  $f(x) = x^3 + 8$ . Indicate which algebraic formula is used.
19. For the function  $f(x) = -x^3 + 2x$ , determine the sign of  $f(x)$  for  $x > 0$  and for  $x < 0$ . Provide a brief explanation with your answer.
20. Analyse  $f(x) = 3x^3 - x + 1$  by creating a brief table of values for  $x = -2, -1, 0, 1$ , and  $2$ . Based on your table, describe the increasing or decreasing trend of the function as  $x$  increases.

## Hard Questions

21. Consider a general cubic function  $f(x) = ax^3 + bx^2 + cx + d$ , where  $a \neq 0$ . Explain how many turning points a cubic function can have at most and justify your answer in a short paragraph.
22. Factorise completely the function  $f(x) = 4x^3 - 12x^2 + 9x$ . State all zeros and indicate any repeated roots with their multiplicities.
23. Determine all values of  $m$  for which  $x = 2$  is a zero of  $f(x) = x^3 + mx^2 - 4x - 4$ . Show all steps leading to your answer.
24. Find a cubic function with integer coefficients that has a double root. Specify your function in its factorised form and clearly state its roots and the multiplicity of each.
25. Use the factor theorem to factorise completely  $f(x) = 2x^3 - 3x^2 - 8x + 12$ . Outline the steps you take to arrive at the factorised form.
26. Explain in a short paragraph why the graph of any cubic polynomial with a non-zero leading coefficient must cross the x-axis at least once.
27. Factorise completely  $f(x) = x^3 - 6x^2 + 11x - 6$ . Then, using pen and paper, sketch its graph. Ensure you label the x-intercepts and the y-intercept.
28. Analyse the transformation of the function  $f(x) = -(x + 2)^3 + 3$ . Explain in detail the effects of the reflection, horizontal shift, and vertical shift compared to  $f(x) = x^3$ .
29. Solve the equation  $x^3 - 2x^2 - x + 2 = 0$  by factorisation by grouping. Write your complete solution process.

30. Discuss the concept of an inflection point for a cubic function in general. Then, for the function  $f(x) = 3x^3 + x - 5$ , determine the coordinates of its inflection point using the standard form property of cubic functions. Provide a clear explanation.