ILLAWARRA TUTORING

In this worksheet you will explore cubic functions and their graphs, learning how they differ from linear and quadratic functions. You will practise evaluating, factorising, and analysing the properties of cubic functions including their turning points, inflection points and end behaviours.

Easy Questions

1. Write down the general form of a cubic function.
2. Evaluate $f(x) = 2x^3 - 3x^2 + x - 5$ at $x = 2$.
3. What is the degree of $f(x) = -3x^3 + x + 4$?
4. State the maximum number of turning points that a cubic function can have.
5. In one or two sentences, describe one key visual difference between the graph of a cubic function and the graph of a quadratic function.

Intermediate Questions

6. Factorise $x^3 - 3x^2 - 4x + 12$ completely.
7. Describe the end behaviour of $f(x) = -x^3 + 5$ as x tends to infinity and negative infinity.
8. Calculate $f(-1)$ for $f(x) = x^3 + 2x^2 - x + 4$.
9. Explain what it means for $x = 3$ to be a root of $f(x) = ax^3 + bx^2 + cx + d$.
10. Find all real roots of $f(x) = x^3 - 6x^2 + 11x - 6$.
11. Sketch the graph of $f(x) = x^3 - 1$.
12. Show that $f(x) = x^3$ has an inflection point at the origin by computing its first and second derivatives.
13. Explain how the graph of $f(x) = x^3$ differs in steepness from that of $g(x) = 2x^3$.
14. Solve the equation $2x^3 - 2 = 0$ for x.
15. Find the coefficients a, b, c and d for the cubic function $f(x) = ax^3 + bx^2 + cx + d$ given that $f(0) = 1$, $f(1) = 4$, $f(-1) = -4$ and $f(2) = 17$.
16. Determine the turning points of $f(x) = x^3 - 3x$ by finding its critical values.

17. For $f(x) = x^3 - 3x$, find the intervals on which f is increasing and the intervals on which it is decreasing.

18. Find the equation of the tangent to the graph of $f(x) = x^3$ at $x = 1$.

19. For $f(x) = 2x^3 + 3x^2 - x + 5$, calculate the values of $f(-2)$ and $f(3)$.

20. Explain why any cubic function must have at least one real zero.

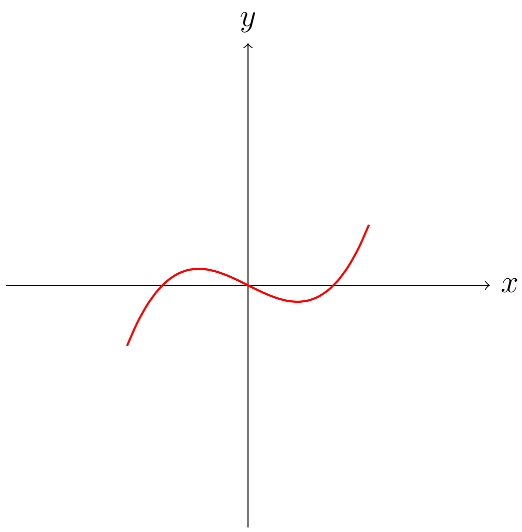
Hard Questions

21. Given $f(x) = x^3 - 6x^2 + 11x - 6$, determine the multiplicity of each of its roots.

22. For a cubic function $f(x) = ax^3 + bx^2 + cx + d$ with an inflection point at (p, q), explain how you would determine the value of p.

23. Find the values of a, b, c and d such that $f(x) = ax^3 + bx^2 + cx + d$ has a local maximum at $(1, 4)$ and an inflection point at $(0, 1)$. (Hint: Use the conditions $f(1) = 4$, $f'(1) = 0$, $f''(0) = 0$ and $f(0) = 1$.)

24. Refer to the diagram below and estimate the coordinates of the inflection point of the cubic function shown.



25. Prove that in the expression $ax^3 + bx^2 + cx + d$, the coefficient a must be nonzero for the function to be cubic.

26. Consider the transformation $g(x) = f(x-2)+3$ where f is a cubic function. Explain how this transformation affects the graph of f.

27. For the cubic function $f(x) = x^3 + px + q$, derive the condition on p and q required for f to have a double (repeated) real root.

28. Assume that the cubic function $f(x)$ has zeros r, s and t. Express $f(x)$ in factored form and explain the relationship between the coefficients and the zeros.

29. Show that if $f(x) = ax^3 + bx^2 + cx + d$ has an inflection point at (h, k), then the function satisfies $f(h + x) + f(h - x) = 2k$ for all x. Explain your reasoning.

30. Consider $f(x) = x^3 - 3x + 2$. Find all of its real zeros and discuss how the presence of any repeated zero affects the graph, particularly in terms of local extrema.