ILLAWARRA TUTORING

In this worksheet you will practise forming and evaluating composite functions. You will learn to combine functions, simplify expressions, and determine any necessary domain restrictions. Answer all questions and show your working.

Easy Questions

1. The functions are defined as $f(x) = 2x + 3$ and $g(x) = x - 1$. Evaluate $f(g(5))$.
2. The functions are given by $f(x) = x^2$ and $g(x) = x + 4$. Find the value of $f(g(2))$.
3. Let $f(x) = \sqrt{x}$ and $g(x) = x + 9$. Compute $f(g(0))$. (Hint: First determine $g(0)$, then apply f.)
4. Given $f(x) = x - 5$ and $g(x) = 3x$, form the composite function $g(f(7))$ and simplify your answer.
5. With $f(x) = 2x$ and $g(x) = x + 3$, find the value of $f\Big(g\big(g(1)\big)\Big)$.

Intermediate Questions

6. For $f(x) = x^2$ and $g(x) = 2x + 1$, determine explicit expressions for $f(g(x))$ and $g(f(x))$.
7. Let $f(x) = x + 3$ and $g(x) = 4x$. Solve for x such that $f(g(x)) = g(f(x))$.
8. Given $f(x) = 3x - 2$ and $g(x) = x^2$, evaluate $f(g(-1))$ and $g(f(-1))$.
9. If $f(x) = \sqrt{x}$ and $g(x) = x - 4$, find the domain of $f \circ g$.
10. For $f(x) = \dfrac{1}{x}$ and $g(x) = x - 2$, state the domain of $h(x) = f(g(x))$.
11. Let $f(x) = x + 1$. Calculate $f\Big(f(2)\Big)$.
12. For $f(x) = x^3$ and $g(x) = x - 1$, find $f(g(x))$ and $g(f(x))$.
13. Given $f(x) = 5 - x$ and $g(x) = 2x$, determine the expression for $f(g(x))$.
14. Let $f(x) = |x|$ and $g(x) = x - 3$. Write $f(g(x))$ in simplified form.
15. With $f(x) = -x$ and $g(x) = 2x + 5$, calculate $f(g(x))$ and $g(f(x))$.
16. Let $f(x) = x^2$ and $g(x) = \sqrt{x + 1}$. Compute $f(g(8))$.

17. Given $f(x) = \dfrac{x}{x+1}$ and $g(x) = x + 2$, express $f(g(x))$ in simplest form and state any restrictions.

18. For $f(x) = x^2 + 2$ and $g(x) = \sqrt{x} + 1$, evaluate $f(g(4))$.

19. Let $f(x) = 2x + 1$ and $g(x) = 3 - x$. Write the expression for $f(g(x))$ and find $f(g(2))$.

20. If $f(x) = \dfrac{1}{x-1}$ and $g(x) = 2x+3$, form the composite function $f(g(x))$ and specify its domain.

Hard Questions

21. Let $f(x) = \dfrac{3x}{x+1}$ and $g(x) = \dfrac{x-2}{x+3}$. Find and simplify the expression for $f(g(x))$.

22. For $f(x) = \dfrac{x+2}{x-1}$ and $g(x) = x - 4$, determine $g(f(x))$ and state any restrictions on x for which the expression is defined.

23. Let $f(x) = x^2 - 1$ and $g(x) = 3x + 2$. Solve for x if $f(g(x)) = 2$. (Hint: Write $f(g(x))$ explicitly and solve the resulting quadratic equation.)

24. Given $f(x) = \sqrt{x+3}$ and $g(x) = x^2 - 4$, form $f(g(x))$ and find the condition(s) on x for which it is defined.

25. Let $f(x) = \dfrac{1}{2-x}$ and $g(x) = 5x + 1$.

    (a) Find an expression for $f(g(x))$.

    (b) Solve for x if $f(g(x)) = 1$.

26. With $f(x) = x^2$ and $g(x) = \dfrac{2}{x+1}$, find $f(g(x))$ and state the restrictions on x.

27. Let $f(x) = 2x + 3$ and $g(x) = \dfrac{1}{x-2}$. Compute $f\Big(g\big(f(1)\big)\Big)$. (Hint: Compute $f(1)$ first, then $g(f(1))$, and finally f of that result.)

28. Given $f(x) = x - 6$ and $g(x) = x^3 + 2x$, first determine the expression for $g(f(x))$. Next, solve for x such that $g(f(x)) = 0$.

29. For $f(x) = \sqrt{x}$ and $g(x) = x^2 - 5$, determine $f(g(x))$ and its domain. Then, solve the equation $f(g(x)) = 3$.

30. Let $f(x) = \dfrac{x+1}{x-1}$ and $g(x) = \dfrac{x-2}{x+2}$.

    (a) Find the composite function $f(g(x))$.

    (b) Simplify the result completely.

    (c) Determine all values of x for which the composite function is undefined.