ILLAWARRA TUTORING

This worksheet focuses on finding the axis of symmetry in a parabola. You will practice calculating the line about which a quadratic function is symmetric by using the formula $x = -\frac{b}{2a}$ and by methods such as completing the square and analysing reflective properties.

Easy Questions

1. Find the axis of symmetry for the quadratic function $y = x^2 + 2x + 1$.
2. Find the axis of symmetry for the function $y = x^2 - 6x + 8$.
3. Determine the axis of symmetry for $y = x^2$.
4. Find the axis of symmetry for $y = 2x^2 + 4x + 1$.
5. Find the axis of symmetry for the quadratic $y = -x^2 + 2x - 3$.

Intermediate Questions

11. Find the axis of symmetry for $y = 3x^2 - 12x + 7$.
12. Find the axis of symmetry for $y = -x^2 + 6x - 5$.
13. Find the axis of symmetry for $y = 0.5x^2 - 3x + 4$.
14. Prove that for any quadratic function $y = ax^2 + bx + c$, the axis of symmetry is given by $x = -\frac{b}{2a}$. (Hint: Complete the square to rewrite the quadratic in vertex form.)
15. Two points on a parabola have equal y values. Show that if the x-coordinates of these points are x_1 and x_2, then the axis of symmetry is $x = \frac{x_1 + x_2}{2}$. Verify your result for the points $(2, 5)$ and $(6, 5)$.
16. Find the axis of symmetry for $y = -2x^2 + 4x + 1$.
17. Find the axis of symmetry for $y = x^2 + 10x + 16$ by completing the square.
18. Determine the axis of symmetry for $y = 4x^2 - 16x + 15$.
19. A parabola has its vertex at $(5, -9)$ and passes through $(7, -5)$. Find its axis of symmetry.

20. A quadratic function has its vertex on the line $x = 3$ and passes through the points $(2, 0)$ and $(4, 0)$. Write the function in vertex form and state its axis of symmetry.

21. Find the axis of symmetry for $y = -3x^2 + 9x - 2$.

22. Find the value of k so that the quadratic function $y = 2x^2 - 8x + k$ has its vertex with a y-coordinate of 3.

23. Derive the vertex form of a quadratic function $y = ax^2 + bx + c$ by completing the square, and state the axis of symmetry in terms of a and b.

24. Find the axis of symmetry for $y = -0.5x^2 + 2.5x - 3$.

25. Given that the quadratic function $y = ax^2 + bx + c$ (with $a = 1$ and $b = -8$) has its axis of symmetry at $x = 4$ and passes through $(2, 0)$, determine the value of c.

Hard Questions

21. Prove that any quadratic function $y = ax^2 + bx + c$ is symmetric about the line $x = -\dfrac{b}{2a}$ by letting $x = -\dfrac{b}{2a} + t$ and showing that the expression for y is unchanged when t is replaced by $-t$.

22. Explain why reflecting the graph of $y = ax^2 + bx + c$ about the line $x = -\dfrac{b}{2a}$ produces the same graph. Include a brief discussion of symmetry in your response.

23. If a quadratic function has two points (x_1, y_1) and (x_2, y_1), show that the axis of symmetry is $x = \dfrac{x_1 + x_2}{2}$. Verify your answer for the points $(1, 5)$ and $(5, 5)$ on the quadratic $y = x^2 - 6x + 10$.

24. Given that the graph of $y = ax^2 + bx + c$ is symmetric about $x = k$, derive the relationship between b and a that leads to $k = -\dfrac{b}{2a}$.

25. Prove that if the two x-intercepts of a quadratic function are equidistant from the axis of symmetry, then the axis is given by the mean of the intercepts. Provide a general proof.

26. For the quadratic function $y = 5x^2 - 20x + 15$, complete the square to express it in vertex form and state the axis of symmetry.

27. The parabola $y = -2x^2 + 4x + k$ has its vertex on the x-axis. Determine the value of k and the equation of the axis of symmetry.

28. Consider the quadratic function $y = ax^2 + bx + c$. Discuss under what conditions on a and b the axis of symmetry $x = -\dfrac{b}{2a}$ is a rational number. Provide examples to support your explanation.

29. A quadratic function is known to be symmetric about $x = \dfrac{3}{2}$. What relationship must hold between a and b in $y = ax^2 + bx + c$? Demonstrate your answer by finding the axis of symmetry for $y = 2x^2 - 6x + 4$.

30. Find the equation of the axis of symmetry of the quadratic function whose graph passes through the points $(0, 1)$, $(2, -3)$ and $(4, 1)$. (Hint: Note that the points $(0, 1)$ and $(4, 1)$ are symmetric about the axis.)