ILLAWARRA TUTORING

In this worksheet you will develop a solid understanding of logarithms, the inverse of exponential functions, and learn how to work with them. Recall that for any positive numbers a and b (with $b \neq 1$), the logarithm $\log_b(a)$ is defined as the unique number c such that $b^c = a$.

Easy Questions

1. Evaluate $\log_2(8)$.
2. Find the value of $\log_{10}(1000)$.
3. Solve for x in the equation $10^x = 100$.
4. Write in your own words the meaning of $\log_b(a) = c$.
5. Determine the value of $\log_3(27)$.

Intermediate Questions

6. Evaluate $\log_5(125)$.
7. Simplify $2^{\log_2(7)}$.
8. Explain why if $b^x = a$ then $\log_b(a) = x$.
9. Solve for x if $\log_3(x) = 4$.
10. Rewrite $\log_7(49) = x$ in exponential form and determine x.
11. Evaluate $\log_{10}(10000)$.
12. Determine $\log_4(16)$.
13. Write the definition of a logarithm and illustrate it by evaluating $\log_2(32)$.
14. Solve for x in the equation $\log_3(x) = 3$.
15. Explain why the logarithm function is the inverse of the exponential function.
16. Evaluate $\log_{100}(10000)$.
17. If $\log_b(81) = 4$, find b by writing $b^4 = 81$.
18. Find $10^{\log_{10}(a)}$ in terms of a for any positive a.

19. Verify that $\log_2(2) = 1$ and explain why this holds true.

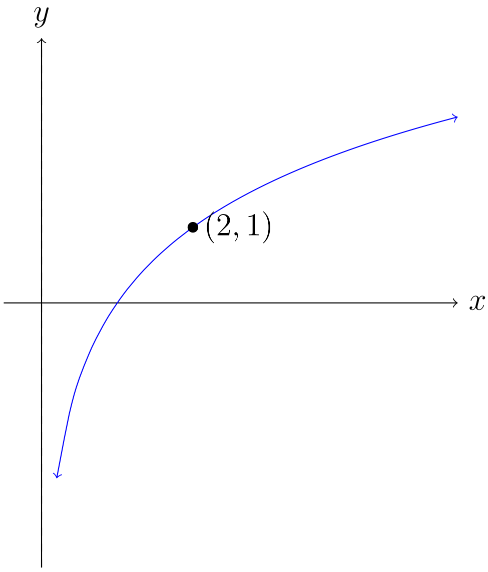
20. Convert the equation $b^y = x$ into logarithmic form.

Hard Questions

21. Explain why the one-to-one nature of the function $f(x) = b^x$ (with $b > 0$ and $b \neq 1$) implies that $\log_b(x)$ is also one-to-one.

22. If $\log_b(16) = 4$, determine b.

23. Examine the diagram below and identify the vertical asymptote of the function $y = \log_2(x)$.



24. Describe how the function $y = \log_{10}(x)$ behaves as x increases and as x approaches 0. Explain the difference in growth rates.

25. Provide an example of a real-life situation that is modelled using a logarithmic scale and briefly explain how the properties of logarithms make them useful in that context.

26. Demonstrate with a numerical example that $10^{\log_{10}(7)}$ equals 7.

27. For the function $f(x) = \log_b(x)$ where $b > 1$, describe how $f(x)$ changes as x increases. Is the function increasing or decreasing? Explain your answer.

28. Explain why the domain of $y = \log_b(x)$ is $(0, \infty)$.

29. Solve the equation $\log_7(x) = 2$ by rewriting it in exponential form.

30. Explain why the logarithm function is the inverse of the exponential function by considering the compositions $f(g(x))$ and $g(f(x))$ where $f(x) = \log_b(x)$ and $g(x) = b^x$.