ILLAWARRA TUTORING

In this worksheet you will develop a solid understanding of logarithms, the inverse of exponential functions. You will learn to convert between logarithmic and exponential forms, interpret the meaning of a logarithm and explore key properties that follow directly from the definition.

## Easy Questions

1. Evaluate $\log_{10}(100)$.
2. Write the exponential form of the statement $\log_2(8) = 3$.
3. Express $\log_5(125)$ as a single number.
4. Find the inverse function of $f(x) = 10^x$.
5. Provide the definition of a logarithm in your own words.

## Intermediate Questions

6. Solve for $x$ if $\log_4(x) = 3$.
7. Write the equivalent exponential form for $\log_7(y) = 2$.
8. Evaluate $\log_3(81)$.
9. Given that $\log_5(125) = 3$, explain in exponential form why this is true.
10. Let $f(x) = \log_2(x)$. Calculate $f(16)$.
11. Sketch the graph of $y = \log_2(x)$ on a coordinate plane. Plot at least three points by converting between exponential and logarithmic forms. (Use pen and paper.)
12. Explain in a short paragraph why logarithms are the inverses of exponential functions.
13. For the function $f(x) = \log_5(x)$, determine the value of $f(25)$.
14. Find $x$ if $\log_{10}(x) = 4$.
15. State the domain of the function $\log_3(x)$.
16. Define the inverse function of $g(x) = \log_2(x)$ in exponential terms.
17. If $f(x) = \log_4(x)$, determine $f(1)$ and $f(4)$.

18. Write a brief explanation of the relationship between the graphs of $y = 4^x$ and $y = \log_4(x)$.

19. Convert the statement "Two to the power of three is eight" into logarithmic form.

20. Explain the significance of the base in a logarithmic expression and illustrate your answer using an example with base 2.

# Hard Questions

21. Prove that the function $f(x) = \log_2(x)$ is one-to-one by showing that if $f(a) = f(b)$ then $a = b$.

22. For the function $f(x) = \log_3(x)$, determine the x-intercept of its graph.

23. Explain how the graph of $y = \log_{10}(x)$ can be used to solve the equation $\log_{10}(x) = 2$.

24. Discuss how varying the base affects the steepness of the logarithmic curve. In your explanation, compare the cases for base 2 and base 10.

25. Show that $\log_5(1) = 0$ by converting the logarithmic equation to its equivalent exponential form.

26. Discuss why the logarithm function is undefined for values of $x \leq 0$. Use the definition of a logarithm in your explanation.

27. If $\log_2(x) = y$, express $x$ in terms of $y$ and explain the process used.

28. Using the concept of logarithms as the inverse of exponentials, explain how one might solve for time in a continuous growth scenario. (Conceptual explanation only.)

29. Describe how the domain and range of $y = \log_7(x)$ compare to those of $y = 7^x$.

30. Create a real-world scenario where logarithms are utilised and explain why the inverse nature of logarithms is vital in that context.