In this worksheet you will investigate the properties and graphs of logarithmic functions, linking them back to exponential models.

Easy Questions

1. State the definition of a logarithmic function. In your answer, explain that for $\log_b(x) = y$, we have $x = b^y$.
2. Evaluate $\log_{10}(10)$.
3. Compute $\log_3(27)$.
4. Determine the value of $\log_5(1)$.
5. For a logarithmic function $\log_b(x)$ with $b > 0$ and $b \neq 1$, state its domain.

Intermediate Questions

6. Sketch the graph of $y = \log_2(x)$ on graph paper. Label the vertical asymptote and the point where the graph crosses the x-axis.
7. Find the x-intercept of $y = \log_3(x)$ and explain your reasoning.
8. Determine the point at which $y = \log_b(x)$ intersects the x-axis.
9. Explain why the graph of $y = \log_b(x)$ has a vertical asymptote at $x = 0$.
10. Given the exponential function $y = b^x$, write its inverse function and state the domain and range of the inverse.
11. Describe the transformations in the function $y = \log_{10}(x - 2) + 3$ compared to $y = \log_{10}(x)$. Specify the horizontal and vertical shifts.
12. Verify that the point (4, 1) lies on the graph of $y = \log_4(x)$ and state the corresponding point on the graph of the inverse exponential function $y = 4^x$.
13. Explain why the graph of $y = \log_2(-x)$ is not simply the reflection of $y = \log_2(x)$ in the y-axis, with reference to the domain.
14. Solve the equation $\log_2(x) = 3$.
15. Determine the domain and range of the function $y = \log_2(x - 1)$.
16. For the function $y = \log_3(x)$, calculate y when $x = 9$.

17. Given the function $y = \log_5(x+4)$, determine the new domain after the horizontal translation.

18. Compare the graphs of $y = \log_2(x)$ and $y = \log_2(2x)$. Explain how multiplying the argument by 2 affects the graph.

19. Describe the effect on the graph of $y = \log_2\left(\frac{x}{2}\right)$ compared to $y = \log_2(x)$, with special attention to the horizontal scaling.

20. Given the point (8, 3) on the graph of $y = \log_2(x)$, verify its consistency with the logarithmic relationship.

Hard Questions

21. Explain how the graph of $y = \log_b(x)$ can be derived as the reflection of $y = b^x$ in the line $y = x$. Include a brief explanation of the inverse function concept.

22. Prove that the graph of $y = \log_b(x)$ is a reflection of the graph of $y = b^x$ with respect to the line $y = x$.

23. For the function $y = \log_2(x-3) - 1$, list the sequence of transformations applied to $y = \log_2(x)$, and describe the effect of each transformation on the graph.

24. Consider the function $y = \log_3(|x|)$. Discuss its graph, including the domain and any symmetry properties.

25. Sketch the graph of $y = -\log_2(x)$ (using pen and paper) and describe the effect of the negative sign on the graph of $y = \log_2(x)$.

26. Explain why the function $y = \log_b(x)$ is one-to-one. Your explanation should incorporate the concept of inverse functions.

27. Compare the graph of $y = \log_{1/2}(x)$ with that of $y = \log_2(x)$. Describe the differences in their shapes and explain why these differences occur.

28. Given the functions $y = \log_3(x)$ and $y = \log_3(x-2)$, determine the horizontal distance between their vertical asymptotes and explain its significance.

29. For the function $y = \log_4(x+1)$, determine the coordinates of the point where the graph crosses the line $y = 0$. Explain your reasoning.

30. Discuss the behaviour of $y = \log_b(x)$ as $x \to 0^+$ and as $x \to \infty$. In your discussion, comment on the rate at which the function increases (or decreases).