ILLAWARRA TUTORING

In this worksheet you will investigate the properties and graphs of logarithmic functions and link them to their corresponding exponential forms. You will work with definitions, domains, ranges, transformations and the inverse relationship between logarithmic and exponential functions.

Easy Questions

1. Identify the value of $\log_b(1)$ for any valid base b.
2. Write the exponential form of the equation $\log_b(x) = y$.
3. Evaluate $\log_{10}(100)$.
4. State the domain and range of the function $y = \log_{10}(x)$.
5. Identify the vertical asymptote of the function $y = \log_b(x)$.

Intermediate Questions

6. Convert the logarithmic equation $\log_4(16) = x$ into exponential form and solve for x.
7. Evaluate $\log_4(16)$.
8. Write down the domain and range of the function $y = \log_2(x)$.
9. Solve for x if $\log_3(x) = 2$.
10. Describe the transformation of the graph of $y = \log_3(x-2)$ compared to $y = \log_3(x)$.
11. Show that $y = \log_3(x)$ is the inverse of $y = 3^x$ by computing the compositions $3^{\log_3(x)}$ and $\log_3(3^x)$.
12. Complete the following table for $y = \log_2(x)$ using $x = 0.5$, 1, 2, 4, and 8. List the corresponding y values.
13. Determine whether the function $y = \log_{10}(x)$ has a y-intercept. Explain your reasoning.
14. Identify the vertical asymptote of the function $y = \log_5(x)$.
15. Write the equation of the graph obtained by shifting $y = \log_4(x)$ vertically upward by 3 units.

16. Sketch, on pen and paper, the graph of $y = \log_2(x + 1)$ and describe its horizontal shift relative to $y = \log_2(x)$.

17. Explain, in plain text, why the function $y = \log_a(x)$ is one-to-one.

18. Compare $y = \log_3(x)$ and $y = \log_3(x - 4)$. State the horizontal displacement between their graphs.

19. Plot several points for the function $y = \log_2(x)$ (choose at least four different values for x) and discuss the trend you observe with respect to increasing values of x.

20. Given the exponential function $y = 2^x$, write its inverse function and explain the relationship between their graphs.

Hard Questions

21. For the function $y = \frac{1}{2}\log_2(x - 1) + 2$, determine the domain, the equation of the vertical asymptote, and any intercepts.

22. Explain, with reference to reflections, how the graph of $y = \log_2(x)$ relates to the graph of its inverse $y = 2^x$.

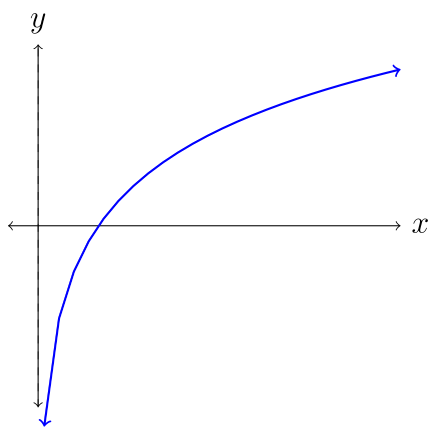
23. Find the inverse of the function $y = \log_3(2x - 5)$. In your answer, specify the domain and range of the inverse function.

24. For $f(x) = \log_4(x + 2) - 1$, find the x-intercept of the graph.

25. For the function $y = \log_5(x - 3) + 2$, determine the coordinates of the point where the graph crosses the horizontal line $y = 2$.

26. Prove algebraically that the function $y = \log_b(x)$ is strictly increasing when $b > 1$.

27. The diagram below shows the graph of $y = \log_2(x)$. Using the diagram, label the vertical asymptote and one clearly defined point on the graph.



28. Given $f(x) = \log_3(x)$, determine its inverse function, $f^{-1}(x)$, and sketch both graphs on the same set of axes (using pen and paper) to show that they are reflections through the line $y = x$.

29. Compare the graphs of $y = \log_2(x)$ and $y = \log_2(10x)$. Describe the effect of the horizontal scaling on the graph.

30. Show that the logarithmic function $y = \log_a(x)$ has no x-intercept and justify your answer with reference to its definition and domain.