ILLAWARRA TUTORING

This worksheet focuses on investigating the properties and graphs of logarithmic functions and linking them back to exponential models. In doing so, you will review the conversion between exponential and logarithmic forms, transformations of logarithmic graphs, and the inverse relationships between logarithmic and exponential functions.

Easy Questions

1. Write $10^2 = 100$ in logarithmic form.
2. State the domain and range of the function $\log_{10}(x)$.
3. Sketch on paper the graph of $y = \log_2(x)$. Clearly label the point where the graph intersects the x-axis.
4. Evaluate $\log_3(27)$.
5. In a few sentences, explain in words how logarithmic functions relate to exponential functions.

Intermediate Questions

6. Find the inverse function of $f(x) = \log_2(x)$ and verify that $f^{-1}(x) = 2^x$.
7. Rewrite the function $f(x) = \log_5(x - 3)$ in words to describe the transformation from the basic function $\log_5(x)$.
8. Sketch on paper the graph of $y = \log_3(x - 2)$. Identify the vertical asymptote of this graph.
9. Determine the x-intercept of the function $y = \log_4(x)$.
10. Describe in words how the graph of $y = \log_2(x)$ is transformed to give the graph of $y = \log_2(x) + 3$.
11. Explain the relationship between the function $f(x) = \log_{10}(x)$ and its corresponding exponential form.
12. Given that the function $y = \log_b(x)$ has a vertical asymptote at $x = 0$, explain what this asymptote represents in terms of the corresponding exponential function.
13. Find the value of x for which $\log_2(x) = 0$.

14. Sketch on paper the graph of $y = \log_3(x+1)$. Ensure you label the vertical asymptote.

15. For the function $f(x) = \log_4(x)$, plot a rough graph on paper and comment on the behaviour as x approaches 0 and as x becomes very large.

16. Compare the graphs of $y = \log_2(x)$ and $y = \log_2(x-2)$. Describe the differences in their domains and x-intercepts.

17. Explain in words how the steepness of the graph changes when comparing $y = \log_2(x)$ and $y = \log_{10}(x)$.

18. Determine the inverse of the function $f(x) = \log_3(x-4)$ and state the domain of the inverse.

19. Explain the effect of reflecting the graph of $y = \log_5(x)$ in the y-axis. Write the equation of the new function and discuss its domain.

20. Given $f(x) = \log_2(x)$, calculate $f(8)$ and then find $f^{-1}(3)$.

Hard Questions

21. Consider the function $f(x) = \log_2(x+3)$. Derive its inverse function $f^{-1}(x)$ and show algebraically that $(f \circ f^{-1})(x) = x$ for all relevant x.

22. For the function $f(x) = \log_5(x)$, show algebraically that $f(1) = 0$ and explain the significance of this in relation to its exponential counterpart.

23. Given $f(x) = \log_7(x+1)$ and $g(x) = \log_7(2x-1)$, find the x value at which their graphs intersect and state the corresponding y-coordinate.

24. Analyse the rate of growth of the function $f(x) = \log_2(x)$ compared to a linear function. In your answer, discuss why the logarithmic function increases at a decreasing rate as x becomes larger.

25. Consider the function $f(x) = 2 + \log_3(x-4)$. Describe in detail how its graph is obtained from the basic graph of $\log_3(x)$, and clearly identify the vertical asymptote and the x-intercept after transformation.

26. Given the function $f(x) = \log_4(x)$, if its graph is reflected over the line $y = x$, determine the equation of the resulting function and describe its domain.

27. For $f(x) = \log_2(x)$, compute $f(16)$ and then explain what this result indicates when considering the inverse exponential function.

28. If the functions $y = \log_3(x)$ and $y = \log_3(2x)$ are graphed on the same axes, determine whether their graphs intersect. Justify your answer.

29. Discuss the significance of the vertical asymptote at $x = 0$ in the graph of $\log_b(x)$ (for any base $b > 1$). Link your discussion to the properties of the corresponding exponential function.

30. Describe a real-world scenario where logarithmic functions serve as the inverse of an exponential model. Explain how the properties of logarithms make them suitable for this application.