ILLAWARRA TUTORING

This worksheet focuses on mastering the laws of logarithms. You will learn to simplify and manipulate logarithmic expressions by applying the product, quotient and power laws. Work through the questions in order and show clear steps in your workings.

Easy Questions

1. Simplify the expression $\log(xy)$ using the logarithm product law.
2. Simplify the expression $\log\left(\frac{x}{y}\right)$ using the logarithm quotient law.
3. Simplify the expression $\log\left(x^4\right)$ by using the logarithm power law.
4. Write the expression $2\log(x) + \log(y)$ as a single logarithm.
5. Expand the expression $\log\left(\frac{a^3b}{c}\right)$ using the logarithm laws.

Intermediate Questions

6. Expand and simplify $\log\left(\frac{a^3b^{\frac{1}{2}}}{c^2}\right)$ using the logarithm laws.
7. Express $\log\left(x^2\right) + \frac{1}{2}\log\left(y\right) - \log\left(z\right)$ as a single logarithm.
8. Write $\log(a) + 2\log(b) - 3\log(c)$ as a single logarithm.
9. Show that $\log\left(x^n\right)$ equals $n\log(x)$ for any positive x and real number n. Explain briefly why this is true.
10. Write $\log\left(\frac{xy^2}{z^3}\right)$ as a combination of sums and differences of logarithms.
11. Simplify $\frac{1}{2}\log(m) + \log(n) - \log\left(m^2\right)$ to a single logarithmic expression.
12. Expand the expression $\log\left(\frac{a^2b}{\sqrt{c}}\right)$ in terms of individual logarithms.
13. Expand $\log\left(\frac{n^5}{p^2q^3}\right)$ using appropriate logarithm laws.

14. Express $\log(x) + \log(y) - \log(z)$ as one logarithm.

15. Write the expression $3\log(t) - 2\log(t-1)$ as a single logarithm.

16. Expand $\log\left(\dfrac{w^2(w+1)^4}{z^5}\right)$ using logarithm laws.

17. State whether the property $\log(a) + \log(b) = \log(ab)$ is true for all positive a and b. Justify your answer briefly.

18. Given that $\log(4) = 0.6021$ and $\log(5) = 0.6990$, use the logarithm laws to calculate $\log(20)$.

19. Expand the expression $\log\left(\dfrac{\sqrt{x}y^3}{z}\right)$ into a sum and/or difference of logarithms.

20. Simplify $2\log(x) - \log(x^2) + \log(y)$ to its simplest form.

Hard Questions

21. Combine $\log\left(\dfrac{ab^3}{c^2\sqrt{d}}\right)$ into one logarithm by applying the logarithm laws.

22. Expand and simplify the expression $\log\left(\dfrac{x^3y^{\frac{1}{2}}}{z^2}\right)$ using logarithm laws.

23. Prove that $\log\left(\dfrac{a}{b}\right) = \log(a) - \log(b)$ for all positive a and b. Include a brief explanation based on the properties of logarithms.

24. Write the expression $\log\left(\dfrac{x^4}{y^2z}\right)$ in expanded form and then show how it can be recombined into a single logarithm.

25. Simplify $\log\left(\dfrac{x^2y}{z^3}\right) + \log(z)$ to a single logarithm.

26. Simplify $\dfrac{1}{2}\left[\log(a) - \log(b)\right] + 2\log(cd) - \log(f^3)$ into one logarithmic expression.

27. Expand $\log\left(\dfrac{p^2q^4}{rs^3}\right)$ into individual logarithms and then factor the expression to combine related terms.

28. Derive and explain why the equality $\log\left(a^n\right) = n\log(a)$ holds for any positive a and real number n. Write a brief explanation using the definition of logarithms.

29. Simplify $3\log\left(m^2n\right) - 2\log\left(mn^3\right)$ to its simplest form by applying logarithm laws and explain each step briefly.

30. Combine $\log(x) + \dfrac{1}{2}\log(y) - 3\log(z) + \log\left(w^2\right)$ into one logarithm. Then, re-expand this single logarithm to show its component parts and simplify fully.