ILLAWARRA TUTORING

This worksheet explores the properties and behaviours of exponential functions and demonstrates how they can be used to model growth and decay. Work through the questions carefully, showing all steps in your reasoning.

## Easy Questions

1. Evaluate $2^3$.
2. Find the y-intercept of $f(x) = 3^x$. (Recall that the y-intercept occurs when $x = 0$.)
3. Evaluate $\left(\frac{1}{2}\right)^2$.
4. Determine whether the function $f(x) = 5^x$ represents exponential growth or decay.
5. Decide if $f(x) = \left(\frac{1}{3}\right)^x$ is a model of growth or decay. Explain your answer.

## Intermediate Questions

6. Write the general form of an exponential function that models growth.
7. A population of bacteria doubles every hour. If the initial population is $P_0$, write an exponential expression that models this growth.
8. A radioactive substance decays such that its quantity is halved every 5 years. Write an exponential model for the decay if the initial amount is $Q_0$.
9. For $f(x) = 7^x$, calculate $f(2)$ and $f(-1)$.
10. Determine the domain and range of the exponential function $f(x) = 4^x$.
11. Describe the behaviour of $f(x) = a^x$ as $x \to \infty$ when $a > 1$.
12. Explain how changing the base affects the behaviour of an exponential function, particularly distinguishing between bases greater than one and between zero and one.
13. Verify that $f(2) = 9$ for $f(x) = 3^x$ is consistent with the function's definition.
14. Describe the transformation applied in $g(x) = 2^{x-1}$ compared with $f(x) = 2^x$.
15. Plot the exponential function $f(x) = 2^x$ by determining at least five key points. (Use pen and paper for your graph.)

16. Determine the equation of an exponential function that passes through $(1, 3)$ and $(3, 27)$.

17. If an exponential function $f(x) = a^x$ passes through the points $(2, 8)$ and $(4, 64)$, find the value of $a$.

18. For $f(x) = 2^x$, evaluate $f(-2)$.

19. Discuss how increasing the base in an exponential function affects the rate of growth.

## Hard Questions

20. A population of 200 bacteria grows exponentially to 800 over 4 hours. Determine the growth factor and write an explicit model for the population.

21. Solve the equation $2^x = 32$.

22. A process follows the model $N(t) = N_0\, a^t$. Given that $N(3) = 5N_0$, determine the value of $a$.

23. For $f(x) = 5^x$, find a value of $x$ such that $f(x)$ is between 50 and 150. Explain your reasoning.

24. A radioactive substance has an initial mass of 100 grams and decays to 50 grams in 7 years. Determine the decay factor and write the corresponding exponential model.

25. Let $f(x) = b^x$ be an exponential function. If $f(1) = 6$ and $f(3) = 216$, find the value of $b$.

26. Consider the function $f(x) = 3^x$. Explain why this function never touches the x-axis.

27. For $f(x) = 2^x$, estimate the value of $f(1.5)$ using reasoning based only on properties of exponential functions.

28. A sum of money doubles every 10 years when invested with continuous growth. Write an expression for this growth and determine the approximate annual growth rate.

29. Compare the graphs of $f(x) = 2^x$ and $g(x) = 2^{x+2}$ by describing the effect of the horizontal shift on the position of the graph.