ILLAWARRA TUTORING

In this worksheet you will explore the properties and behaviours of exponential functions and see how they model growth and decay. You will work through questions of increasing difficulty. Remember that all expressions containing mathematics should be interpreted in displaystyle within inline math mode, for example, $f(x) = a^x$.

Easy Questions

1. Evaluate $f(x) = 2^x$ at $x = 3$. Write your answer as a single number.
2. State the domain and range of the exponential function $f(x) = 3^x$.
3. Write a brief explanation of why $f(x) = a^x$ (with $a > 0$ and $a \neq 1$) is called an exponential function.
4. Write an exponential model for a population that doubles every unit time, assuming an initial population of P_0.
5. Decide whether $f(x) = \left(\dfrac{1}{2}\right)^x$ is increasing or decreasing and explain your reasoning in one sentence.

Intermediate Questions

6. Evaluate $f(x) = 2^x$ at $x = 0$, $x = 2$, and $x = -3$. Show your working.
7. Sketch the graph of $f(x) = 3^x$ on pen and paper. Label the y-intercept and the horizontal asymptote.
8. A certain bacteria population triples every 4 hours. If the initial count is P_0, write an exponential function $P(t)$ that models the population after t hours.
9. A radioactive substance decays such that every 3 years its mass halves. If the initial mass is M_0, write an exponential model $M(t)$ to represent the decay.
10. Given that $f(x) = 5 \cdot a^x$ and that $f(0) = 5$ and $f(2) = 20$, find the value of a. Show your working.
11. For the function $f(x) = 4^x$, write down the domain and range.
12. An investment grows by 8% each year. Write an exponential model for the value $V(t)$ after t years given an initial investment $V(0)$. Explain each part of your model.

13. For the function $f(x) = \left(\frac{1}{2}\right)^x$, calculate $f(1)$ and $f(-1)$.

14. Express $f(x) = 8^x$ in the form $f(x) = 2^{kx}$ by finding the appropriate constant k.

15. For the function $f(x) = 10^x$, determine the multiplicative factor by which the function value increases when x increases by 1. Explain your answer.

16. Determine whether the function $f(x) = 0.7^x$ is increasing or decreasing and provide a short explanation.

17. The function $g(x) = 2^{(x-3)}$ is a horizontal shift of $f(x) = 2^x$. Describe the effect of this transformation on the graph of $f(x)$.

18. Explain how the graph of $f(x) = 2^x$ changes when it is transformed to $g(x) = 4 \cdot 2^x$. What effect does the factor 4 have?

19. An object cools down following an exponential decay model. If its temperature at time t is given by $T(t) = T_a + (T_0 - T_a)b^t$, where T_a is the ambient temperature and $0 < b < 1$, explain the role of b and what it indicates about the cooling process.

20. For a typical exponential function $f(x) = a^x$ (with $a > 1$), explain the significance of the horizontal asymptote. In your answer, indicate its position and relevance in modelling.

Hard Questions

21. Prove that an exponential function $f(x) = a^x$, where $a > 0$ and $a \neq 1$, is continuous and monotonic. Write a short explanation based on the definition of continuity and the properties of the exponential function.

22. A sum of money is invested and doubles every 6 years. Write an exponential growth model for the investment and then determine the approximate growth factor for one year. Explain your reasoning.

23. Consider the function $f(x) = 2^x$. Describe the transformation that yields the function $g(x) = -2^x + 5$. In your answer, state the effects of reflection, vertical shift, and any other modifications.

24. Without using logarithms, compare the functions $f(x) = 3^x$ and $g(x) = 5^x$ by explaining which grows faster and why. Use at least one numerical example in your explanation.

25. Given the function $f(x) = \left(\frac{1}{3}\right)^{(x-2)} + 4$, determine the horizontal asymptote and describe how the graph is shifted relative to the basic function a^x. Support your answer with a description of the effect of each transformation.

26. Carbon-14 has a half-life of approximately 5730 years. Write an exponential decay model for the amount $A(t)$ of Carbon-14 remaining after t years, assuming an initial amount A_0. Explain each parameter in your model.

27. A population triples every 30 years. Write an exponential model for the population and then determine, using your model, the approximate percentage increase in one year. Explain each step in your reasoning.

28. Derive and explain why for any real numbers x and y, the exponential function satisfies $a^{x+y} = a^x \cdot a^y$. Provide a clear, step-by-step explanation.

29. Solve for x in the equation $2^{(x+1)} = 16$. Explain how rewriting 16 as a power of 2 leads to the solution.

30. Discuss the end behaviour of the exponential function $f(x) = a^x$ for $a > 1$. In your discussion, describe what happens as x approaches positive infinity and negative infinity, and explain the implications of this behaviour in modelling real-world growth and decay.