ILLAWARRA TUTORING

This worksheet focuses on exploring the properties and behaviours of exponential functions. In these questions you will review the definition, key features and applications of exponential functions for modelling growth and decay. Remember that an exponential function is generally written in the form $f(x) = a \cdot b^x$ where $a \neq 0$ and $b > 0$, $b \neq 1$.

Easy Questions

1. Write down a definition of an exponential function and give one example.
2. Evaluate $f(x) = 2^x$ for $x = 0$, $x = 1$ and $x = 2$.
3. Find the y-intercept of the function $f(x) = 3 \cdot 2^x$.
4. Using pen and paper, sketch a rough graph of $f(x) = 2^x$. Label the y-intercept and indicate the direction of growth.
5. State the domain and range of the exponential function $f(x) = 5^x$.

Intermediate Questions

6. Consider the function $f(x) = 4 \cdot \left(\frac{1}{2}\right)^x$. Evaluate $f(3)$ and state whether the function represents growth or decay.
7. Determine the equation of the exponential function which has a y-intercept of 3 and doubles in value for each unit increase in x.
8. A radioactive substance decays so that its quantity halves every 4 hours. Write an expression in the form $f(t) = a \cdot b^t$ to model this decay.
9. Solve for x if $2^{x+1} = 2^3$.
10. Find the exponential function $f(x) = a \cdot b^x$ given that $f(0) = 10$ and $f(2) = 40$.
11. For the function $f(x) = 3 \cdot (1.5)^x$, calculate $f(0)$, $f(1)$ and $f(2)$. Then, list these values in a table.
12. Describe qualitatively what happens to the function $f(x) = 5 \cdot (0.8)^x$ as x increases.
13. Identify the horizontal asymptote of the function $f(x) = 2^x$.
14. For the function $f(x) = 7 \cdot 3^x$, find the value of $f(-1)$.

15. Compare the functions $f(x) = 2^x$ and $g(x) = 2^{x+3}$. Describe the transformation that relates $g(x)$ to $f(x)$.

16. Explain the effect of changing the parameter a in the exponential function $f(x) = a \cdot 2^x$.

17. A population of 200 bacteria doubles every 3 hours. Write an exponential model for the population and determine the population after 6 hours.

18. Consider the exponential function $f(x) = a \cdot b^x$. Explain what can be said about the behaviour of $f(x)$ if $b > 1$ compared to when $0 < b < 1$.

19. Show that for any exponential function $f(x) = a \cdot b^x$, the value $f(0)$ equals a.

20. Describe a real-world scenario that can be modelled by an exponential growth function. Explain why an exponential model is appropriate.

Hard Questions

21. Prove that any exponential function $f(x) = a \cdot b^x$ (with $a \neq 0$ and $b > 0$, $b \neq 1$) is one-to-one.

22. A bacteria population increases by 50 per cent every hour. If the initial population is 200, find an expression for the population after t hours.

23. Consider the exponential function $f(x) = a \cdot b^x$. If we write b as $1 + r$ for some rate r, explain how the value of r determines the nature of the function.

24. Prove that for the function $f(x) = a \cdot b^x$, the ratio $\dfrac{f(x+1)}{f(x)}$ is constant and equals b.

25. For any exponential function $f(x) = a \cdot b^x$ with $b > 1$ or $0 < b < 1$, demonstrate that there is a horizontal asymptote. State the asymptote.

26. Given $f(x) = 8 \cdot \left(\dfrac{1}{2}\right)^x$, calculate $f(-2)$.

27. Determine if the function $f(x) = -3 \cdot 2^x$ satisfies the criteria for being an exponential function. Explain your reasoning.

28. Describe in words and with a hand-drawn diagram (using pen and paper) how the graph of $f(x) = 5 \cdot 3^{x-2}$ is related to the parent function 3^x. Include details of any shifts or stretches.

29. Consider two functions $f(x) = c \cdot d^x$ and $g(x) = c \cdot d^{x+k}$ where k is a constant. Explain the effect of the parameter k on the graph of $g(x)$ compared to $f(x)$.

30. A sum of money P is invested at an interest rate r per period under compound interest. Derive an expression for the amount after n periods in the form $A = P \cdot (1 + r)^n$. State any assumptions made in deriving the model.