ILLAWARRA TUTORING

This worksheet focuses on differentiation of exponential functions. You will practise applying rules for differentiating functions of the form $e^{g(x)}$, including use of the chain, product and quotient rules. Solve rate of change problems where an exponential model is given. Remember to show all your working.

Easy Questions

1. Differentiate the function $f(x) = e^x$.

2. Differentiate the function $f(x) = e^{3x}$.

3. Differentiate the function $f(x) = 5e^x$.

4. Differentiate $f(x) = e^{2x}$ and evaluate the derivative at $x = 0$.

5. Differentiate the function $f(x) = e^{x+2}$.

Intermediate Questions

6. Differentiate $f(x) = e^{x^2}$.

7. Differentiate $f(x) = e^{3x+2}$.

8. Differentiate $f(x) = e^{\sin x}$.

9. Differentiate the function $f(x) = 4e^{2x} - 3e^x$.

10. Differentiate $f(x) = e^x \cdot e^{2x}$ after simplifying the expression.

11. Differentiate $f(x) = x\,e^x$ using the product rule.

12. Differentiate the function $f(x) = e^{2x} + e^{3x}$.

13. Differentiate $f(x) = x\,e^{2x^2}$.

14. A population grows according to $P(t) = P_0 e^{rt}$, where P_0 and r are constants. Differentiate $P(t)$ with respect to t.

15. Differentiate $f(x) = 3e^{2x}$ and then find the second derivative $f''(x)$.

16. Compute the second derivative of $f(x) = e^{x}$.

17. Differentiate $f(x) = e^{5-2x}$.

18. Differentiate $f(x) = 7e^{2x} + 4x\,e^{2x}$.

19. An object cools according to $T(t) = T_0 e^{-kt}$, where T_0 and $k > 0$ are constants. Differentiate $T(t)$ with respect to t.

20. Differentiate the function $f(x) = (e^{x})^{2}$, simplifying the expression before differentiating.

Hard Questions

21. Differentiate $f(x) = e^{3x^2+2x-1}$.

22. Differentiate $f(x) = e^{x^3}$.

23. Using the limit definition of the derivative, prove that if $f(x) = e^{g(x)}$, then $f'(x) = g'(x)e^{g(x)}$.

24. Differentiate $f(x) = \dfrac{e^{x}}{x}$ using the quotient rule.

25. Simplify the function $f(x) = x\,e^{x}\,e^{2x}$ and then differentiate your simplified expression.

26. The concentration of a substance is given by $C(t) = C_0 e^{-kt}$, where $k = 0.3$. Differentiate $C(t)$ to find $\dfrac{dC}{dt}$ and determine its value when $t = 5$.

27. Consider the piecewise function

$$f(x) = \begin{cases} e^{x} & \text{if } x \geq 0, \\ 2e^{x} & \text{if } x < 0. \end{cases}$$

Differentiate $f(x)$ on each interval and determine the right-hand and left-hand derivatives at $x = 0$.

28. Differentiate $f(x) = e^{\tan(x)}$ for all x where $\tan(x)$ is defined.

29. Let $f(x) = e^{x} + e^{2x} + e^{3x}$. Differentiate $f(x)$ and compute $f'(0)$.

30. Differentiate $f(x) = e^{\sqrt{x}}$, showing your use of the chain rule and simplifying your answer.