ILLAWARRA TUTORING

In this worksheet you will learn to use the change of base formula to evaluate logarithms in different bases. Recall that $\log_b a = \dfrac{\log_c a}{\log_c b}$ for any positive base $c \neq 1$. Work through the following questions.

Easy Questions

1. Evaluate $\log_2 8$ using the change of base formula with base 10.
2. Evaluate $\log_5 25$ by writing it as a quotient of logarithms in base 10.
3. Evaluate $\log_3 81$ using the change of base formula with natural logarithms.
4. Write $\log_7 49$ in terms of base 10 logarithms using the change of base formula.
5. Evaluate $\log_4 16$ by applying the change of base formula with a suitable base.

Intermediate Questions

6. Evaluate $\log_3 9$ using the change of base formula.
7. Use the change of base formula with base 2 to compute $\log_8 16$.
8. Express $\log_2 5$ as $\dfrac{\log_{10} 5}{\log_{10} 2}$.
9. Write $\log_5 3$ as $\dfrac{\ln 3}{\ln 5}$.
10. Express $\log_7 2$ in terms of common logarithms.
11. Evaluate $\log_{16} 64$ by rewriting it with the change of base formula.
12. Simplify $\dfrac{\ln 125}{\ln 5}$ using the change of base formula.
13. Simplify $\dfrac{\log_{10} 49}{\log_{10} 7}$ using the change of base formula.
14. Express $\log_{x^2} a$ in terms of $\log_x a$.
15. Show that $\dfrac{\log_b a}{\log_b c} = \log_c a$ by using the change of base formula, and verify your result for $a = 8$, $b = 2$ and $c = 4$.

16. Given $\log_k 16$, verify the identity $\log_k 16 = \dfrac{4}{\log_2 k}$ by applying the change of base formula.

17. Evaluate $\log_{0.5} 8$ using the change of base formula.

18. Express $\log_9 27$ in terms of logarithms with base 3 using the change of base formula.

19. Using the change of base formula write $\log_b a = \dfrac{\log_{10} a}{\log_{10} b}$. Then evaluate $\log_5 50$ (you may use a calculator for an approximate value).

20. Prove that $\log_b a = \dfrac{1}{\log_a b}$ by using the change of base formula.

Hard Questions

21. Derive the change of base formula starting from the definition of logarithms. Explain each step in your derivation.

22. Given that $\log_b a = x$, prove that $a = b^x$ and use this result to derive the change of base formula.

23. Solve for x in the equation $\log_x 27 = \dfrac{3}{\log_3 x}$ by rewriting the logarithms using a common base.

24. Given $\log_2 x = b$, express $\log_4 x$ in terms of b using the change of base formula.

25. If $\log_x 8 = \log_2 16$, solve for x by rewriting both sides with a common base.

26. Given $\log_{2x} 32 = 5$, solve for x using the change of base formula.

27. Prove that for any positive numbers a, b and c, the identity $\log_a c = \log_a b \cdot \log_b c$ holds. Use the change of base formula in your proof.

28. If $\log_x y = \dfrac{1}{2}$, express y in terms of x using the definition of logarithms and the change of base formula.

29. Show that $\dfrac{\log_m n}{\log_m p} = \log_p n$ by using the change of base formula. Then, evaluate this expression for $m = 10$, $n = 25$, and $p = 5$ (provide an approximate answer if necessary).

30. Prove that $\log_b \sqrt{a} = \dfrac{1}{2} \log_b a$ by using the change of base formula and explain each step in your proof.