In this worksheet you will learn to use the change of base formula to evaluate logarithms in different bases. You will practice rewriting logarithms with a new base and applying the formula to compute values.

Easy Questions

1. Write the change of base formula for logarithms in your own words.
2. Evaluate $\log_4 16$ using the change of base formula.
3. Evaluate $\log_3 27$ using the change of base formula.
4. Express $\log_5 125$ in terms of common logarithms and simplify.
5. Verify that $\log_a b = \dfrac{1}{\log_b a}$ using the change of base formula.

Intermediate Questions

6. Evaluate $\log_7 49$ using natural logarithms.
7. Express $\log_9 27$ in terms of common logarithms.
8. Solve for x if $\log_x 16 = 4$ by rewriting the equation using the definition of logarithms.
9. Evaluate $\log_8 32$ using the change of base formula.
10. Express $\log_4 32$ in terms of $\log_2$ and simplify.
11. Evaluate $\log_5 625$ using natural logarithms.
12. Show that $\log_a b = \dfrac{1}{\log_b a}$ by applying the change of base formula.
13. Express $\log_2 10$ using common logarithms.
14. Simplify $\dfrac{\log_3 81}{\log_3 9}$ using the change of base formula.
15. Given that $\log_3 7 \approx 1.7712$ and $\log_3 5 \approx 1.4649$, use the change of base formula to calculate $\log_5 7$.
16. Evaluate $\log_6 216$ using natural logarithms.
17. Given $\log_2 5 \approx 2.3219$, find $\log_4 5$ using the change of base formula.

18. Express $\log_{12} 18$ in terms of natural logarithms using the change of base formula.
19. Evaluate $\log_{16} 32$ using the change of base formula.
20. Express $\log_{27} 9$ in terms of $\log_3$ and simplify.

Hard Questions

21. Derive the change of base formula $\log_b a = \dfrac{\log_c a}{\log_c b}$ starting from the definition of logarithms.
22. Prove that $\log_b a = \dfrac{\log_c a}{\log_c b}$ is valid for any positive constant $c \neq 1$ by comparing exponential forms.
23. Solve for x if $\log_x 81 = \dfrac{4}{3}$ by rewriting the expression in exponential form.
24. Express $\dfrac{\ln 64}{\ln 8}$ in the form of a logarithm and simplify the result.
25. Given that $\log_{10} 2 \approx 0.3010$, evaluate $\log_2 50$ by rewriting it in common logarithms.
26. Evaluate $\log_{\sqrt{2}} 8$ using the change of base formula.
27. Simplify $\log_4 2 + \log_2 16$ using the change of base formula and properties of logarithms.
28. Evaluate $\dfrac{\log_2 45}{\log_5 45}$ and simplify your answer using the change of base formula.
29. Prove that $\log_{b^k} a = \dfrac{1}{k} \log_b a$ by applying the change of base formula and logarithmic properties.
30. Given that $\log_2 3 \approx 1.585$, evaluate $\log_8 27$ using the change of base formula.