ILLAWARRA TUTORING

In this worksheet you will learn to use the change of base formula to evaluate logarithms in different bases. You will practise rewriting logarithms using a new base (commonly base 10 or base e) and apply this technique in both numerical and algebraic contexts.

Easy Questions

1. Write the change of base formula for logarithms using common logarithms.
2. Evaluate $\log_2 8$ using common logarithms and the change of base formula.
3. Evaluate $\log_3 9$ using the change of base formula with common logarithms.
4. Express $\log_5 25$ in terms of common logarithms using the change of base formula.
5. Show that $\log_b a = \dfrac{1}{\log_a b}$.

Intermediate Questions

6. Evaluate $\log_4 64$ using the change of base formula with common logarithms.
7. Express $\log_2 5$ in terms of natural logarithms.
8. Write $\log_7 125$ in terms of common logarithms using the change of base formula.
9. Given $\log_{10} 7 \approx 0.8451$, find $\log_7 10$ using the change of base formula.
10. Express $\log_6 36$ using natural logarithms.
11. Evaluate $\log_7 49$ using the change of base formula with natural logarithms.
12. Verify the identity $\log_a b \cdot \log_b a = 1$ using the change of base formula.
13. Express $\log_4 2$ in terms of $\log_2 2$.
14. Solve for x if $\log_3 x = 2$.
15. Determine $\log_5 125$ using the change of base formula with common logarithms.
16. Rewrite $\log_2 10$ in terms of natural logarithms.
17. Use the change of base formula to simplify $\log_9 27$ in terms of natural logarithms.
18. Given $\log_{10} 2 \approx 0.3010$, compute $\log_2 10$.

19. Express $\log_8 32$ using natural logarithms and calculate its approximate value.

20. Derive the inverse relationship between $\log_b a$ and $\log_a b$ using the change of base formula.

Hard Questions

21. Prove the change of base formula starting from the definition of logarithms.

22. Solve the equation $\log_4 x = \dfrac{\log_{10} x}{\log_{10} 4}$ and discuss why this equality holds.

23. Show that $\log_b a = \dfrac{1}{\log_a b}$ and then use this result to compute $\log_2 8$.

24. Derive an expression for $\log_2 60$ using common logarithms given that $\log_{10} 2 \approx 0.3010$, $\log_{10} 3 \approx 0.4771$, and $\log_{10} 5 \approx 0.6990$.

25. Given that $\log_3 2 \approx 0.6309$, evaluate $\log_2 3$ using the change of base formula.

26. Simplify $\dfrac{\log_7 49}{\log_7 7}$ using the change of base concept.

27. If $\log_b a = x$, express $\log_b(a^3)$ in terms of x.

28. Verify that $\log_2 18 = \dfrac{\ln 18}{\ln 2}$ and estimate its value using $\ln 2 \approx 0.6931$ and $\ln 18 \approx 2.8904$.

29. Prove that for any positive real numbers a and b, $\dfrac{\log a}{\log b} = \log_b a$.

30. Solve for x in the equation $\dfrac{\log x}{\log 2} = \dfrac{\log(x+6)}{\log 3}$.