In this worksheet you will learn how exponential functions are used to model real-world scenarios like compound interest, population growth, and radioactive decay. Answer each question carefully and show all your working.

Easy Questions

1. A bacteria culture doubles every 3 hours. If the initial population is 100, calculate the population after 9 hours.
2. A principal of 1000 dollars is invested at an annual interest rate of 5% compounded once per year. Calculate the amount in the account after 3 years using the formula $A = P(1 + \frac{r}{1})^{1 \cdot t}$.
3. A radioactive substance halves in quantity every 10 years. If the initial amount is 80 grams, determine the amount remaining after 30 years.
4. Write the formula for continuous compound interest. Then, using the formula $A = P\,e^{rt}$, calculate the amount after 2 years on a 500 dollar investment at an annual rate of 6%.
5. Evaluate the expression $e^{\ln(5)}$ and explain why the result is as obtained.

Intermediate Questions

6. A bank account has the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$. If $P = 2000$, $r = 0.04$, $n = 4$ (quarterly compounding), and $t = 5$ years, calculate the balance.
7. A population grows continuously as $P(t) = P_0\,e^{rt}$. If the population triples in 10 years, determine the value of r.
8. Radioactive decay is given by $Q(t) = Q_0\,e^{-kt}$ and the half-life is 8 years. Determine the decay constant k.
9. An investment of 1500 dollars is compounded continuously at an annual interest rate of 7%. Determine the time required for the investment to double.
10. A car depreciates continuously at a rate of 15% per year. Write an expression for its value after t years if its current value is 20000 dollars. Then, evaluate the value after 3 years.

11. An initial deposit of 5000 dollars grows to 7500 dollars in 12 years with continuous compounding. Calculate the annual interest rate.

12. The number of bacteria is modelled by $N(t) = 1000\,e^{0.23t}$. Find the number of bacteria after 4 hours, leaving your answer in terms of e.

13. A city's population is given by $P(t) = 500000\,e^{0.015t}$ where t is the number of years since 2000. In which year will the population exceed 600000?

14. An investment account offers 3.5% per annum compounded continuously. Express the account balance after t years if the initial deposit is 1000 dollars, and determine the balance after 20 years.

15. A substance decays so that its mass becomes 80% of its value every 5 years. Write the equivalent expression in the form $m(t) = m_0\,e^{rt}$ and compute the continuous rate of decay r.

16. A cell culture grows according to $C(t) = C_0\,e^{kt}$. If the cell count increases from 120 to 480 in 16 hours, determine the growth rate k.

17. A city's population declines continuously by 2% per year. Express the population after t years if the current population is 1000000. Then, compute the population after 10 years.

18. In a continuously compounded account, $A = P\,e^{rt}$. If an investment of 2500 dollars grows to 4000 dollars in 8 years, determine the annual interest rate r.

19. Find the continuous growth rate required for an exponentially growing process to increase by 50% in 15 years.

20. A bacteria population doubles every 6 hours under continuous growth. Determine the continuous growth rate r and write the function in the form $N(t) = N_0\,e^{rt}$.

Hard Questions

21. A radioactive substance decays as $Q(t) = Q_0\,e^{-kt}$. If 30 grams remain after 5 years from an initial 80 grams, calculate the decay constant k.

22. A bank advertises continuous compounding. If 10000 dollars grows to 13000 dollars in 8 years, set up the equation and compute the annual interest rate.

23. A town's population is modelled by $P(t) = P_0\,e^{rt}$ and increases by 40% over 25 years. Determine the continuous growth rate r.

24. An investment of 5000 dollars compounded continuously at 6% per annum grows to 10000 dollars. Determine the time required for this doubling, expressing your answer in terms of natural logarithms.

25. A radioactive isotope has a half-life of 12 years. Write the expression for the remaining mass after t years and determine the mass remaining after 30 years if the initial mass is 200 grams.

26. A population of insects grows according to $N(t) = N_0 e^{rt}$. A scientist observes that there are 150 insects after 4 days and 600 insects after 8 days. Determine the continuous growth rate r and the initial population N_0.

27. A car depreciates continuously at a rate of 18% per year. With an initial value of 25000 dollars, find the time it will take for its value to fall below 10000 dollars.

28. A cell culture grows to five times its original size in 12 hours under continuous growth. Find the continuous growth rate k and then determine the time required for the culture to grow to seven times its original size.

29. An investment of 800 dollars compounded continuously grows to 2000 dollars in 15 years. Express the annual continuous interest rate in terms of natural logarithms.

30. A medical researcher models the decay of a drug with the formula $C(t) = C_0 e^{-rt}$. If the concentration falls to 20% of its initial value after t minutes, express r in terms of t, and then calculate r when $t = 40$ minutes.