ILLAWARRA TUTORING

In this worksheet you will explore quadratic equations. You will learn to identify and manipulate quadratic equations in standard form, understand the significance of the discriminant, and analyse the properties of their graphs. Read each question carefully and ensure you explain your reasoning where asked.

Easy Questions

1. Write down whether the following equation is quadratic or not: $2x^2 + 3x + 1 = 0$. Explain your answer briefly.
2. Rewrite the equation $(x - 3)^2 = 4$ in the standard quadratic form $ax^2 + bx + c = 0$.
3. For the equation $5x^2 - 2x + 7 = 0$, state the values of a, b, and c.
4. Determine the degree of the equation $x^2 - 4 = 0$ and explain what makes it a quadratic equation.
5. Calculate the discriminant $\Delta = b^2 - 4ac$ of the equation $x^2 + 6x + 9 = 0$, and state the nature of its roots.

Intermediate Questions

6. For the quadratic equation $2x^2 + 3x - 5 = 0$, explain how the value of the discriminant indicates the nature of its solutions without actually solving the equation.
7. For the quadratic $x^2 - 4x + 3 = 0$, describe the effect on the graph if the coefficient of x^2 is increased while the other coefficients remain constant.
8. Show that the axis of symmetry for a quadratic function $ax^2 + bx + c$ is given by $x = -\dfrac{b}{2a}$, explaining your reasoning without solving for the roots.
9. Given the quadratic equation $x^2 + kx + 9 = 0$, determine for which values of k the equation has real roots.
10. Explain in your own words the significance of the discriminant in determining whether a quadratic equation has two distinct real solutions, one repeated solution, or no real solutions.
11. Consider the quadratic equation $3x^2 - 12x + 12 = 0$. Calculate the x-coordinate of its vertex using $x = -\dfrac{b}{2a}$, then find the corresponding y-value by substituting back into the quadratic expression.

12. Compare the graphs of $x^2 + 4x + 3$ and $x^2 + 4x + 5$. Describe how the constant term affects the vertical position of the graph relative to the x-axis.

13. For the quadratic $-x^2 + 2x + 3 = 0$, discuss the orientation of its graph and explain what this implies about the existence of a maximum or minimum value.

14. For the quadratic function $f(x) = x^2 + 8x$, describe how altering the coefficient of x (while keeping the quadratic term unchanged) would affect the horizontal placement of the graph.

15. Consider the quadratic function $f(x) = ax^2 + bx + c$. Discuss how varying the value of a (with b and c fixed) affects the width of the parabola and the location of its vertex.

16. For the quadratic $x^2 - 2x + 1 = 0$, sketch a diagram of its graph on pen and paper and explain why the graph shows that the equation has a repeated real root.

17. Compute the discriminant of $4x^2 - 4x + 1 = 0$ and explain what this reveals about the points where the graph meets the x-axis.

18. Explain how the value of the constant term c in $ax^2 + bx + c$ influences the number of x-intercepts of the graph of the quadratic function.

19. Discuss how changing the sign of the coefficient a in the quadratic equation $ax^2 + bx + c = 0$ affects the direction in which the graph opens.

20. Consider the function $f(x) = x^2 - 2x$. Demonstrate that its minimum value is attained at $x = 1$ and justify your answer.

Hard Questions

21. Prove that any quadratic equation $ax^2 + bx + c = 0$ (with $a \neq 0$) can be rewritten in the form $a\left(x + d\right)^2 + e = 0$ for suitable constants d and e. Explain the process conceptually without necessarily carrying out all algebraic steps.

22. Derive the condition on the coefficients a, b, and c for which the quadratic equation $ax^2 + bx + c = 0$ has no real roots. Explain the implications of this condition for the graph of the quadratic function.

23. Given that the quadratic $x^2 + px + 4$ has two equal real roots, determine the value of p. Justify your answer using the concept of the discriminant.

24. Consider a quadratic function $f(x) = ax^2 + bx + c$. Show that if the function is shifted upward by 5 units (i.e. considering $f(x) + 5$), the discriminant remains unchanged. Provide a clear explanation for your answer.

25. Explain why any quadratic equation of the form $ax^2 + bx + c = 0$ with $a > 0$ has a minimum value. Support your response with reference to either calculus or a graphical explanation.

26. Examine the quadratic equation $2x^2 + (k - 3)x + k = 0$. Determine the range of values for k that guarantees the equation has two distinct real roots. Explain your reasoning in detail using the discriminant.

27. Let $f(x) = ax^2+bx+c$ be a quadratic function. Prove that its vertex $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ is the unique point where the function changes direction, using the idea of symmetry.

28. Establish the relationship between the coefficients a, b, and c and the coordinates of the vertex of the quadratic function $f(x) = ax^2 + bx + c$. Discuss how each coefficient influences the vertex's position in the xy-plane.

29. Analyse the effect of multiplying a quadratic function $f(x) = ax^2 + bx + c$ by a constant factor k. In your answer, discuss how the vertex and the discriminant are affected by this scaling.

30. Given that the quadratic equation $3x^2 + rx + s = 0$ has exactly one real solution, derive a relationship between r and s. Provide a thorough explanation using the concept of the discriminant.