ILLAWARRA TUTORING

This worksheet will help you explore quadratic equations. You will investigate their structure, the nature of their solutions via the discriminant, and the geometry of their graphs. Ensure you understand the reasoning behind each approach.

Easy Questions

1. Write the quadratic equation $3x^2 + 5x = 2$ in standard form.
2. For the quadratic equation $4x^2 - 7x + 3 = 0$, identify the coefficients a, b and c.
3. Calculate the discriminant of the quadratic equation $x^2 + 6x + 9 = 0$.
4. Sketch the graph of $y = x^2$ and label its vertex and axis of symmetry.
5. Determine the axis of symmetry for the quadratic function $y = 2x^2 - 4x + 1$.

Intermediate Questions

6. For the quadratic equation $x^2 - 4x + 3 = 0$, use the discriminant to determine the nature of its roots.
7. Find the vertex of the quadratic function $f(x) = x^2 - 8x + 15$ by using the formula $x = -\frac{b}{2a}$.
8. Rewrite the quadratic function in vertex form: $y = x^2 + 4x + 1$.
9. Describe how the graph of $y = ax^2 + bx + c$ changes when the value of a increases (with other coefficients fixed).
10. Determine the range of the quadratic function $f(x) = 2x^2 - 8x + 10$.
11. Given the quadratic in vertex form $y = -3(x - 2)^2 + 12$, state the vertex and the axis of symmetry.
12. For the equation $x^2 - 4x + 4 = 0$, explain why there is exactly one real solution.
13. A ball is thrown so that its height is given by $h(t) = -5t^2 + 20t + 2$. Determine the time at which the ball reaches its maximum height.
14. Using the ball height function $h(t) = -5t^2 + 20t + 2$, calculate the maximum height reached.
15. Explain why a quadratic equation can have at most two real solutions.

16. Consider the quadratic function $f(x) = x^2 + kx + 9$. Determine for which values of k the equation $f(x) = 0$ has real solutions.

17. For the quadratic $y = x^2 + 4x + c$, describe how varying c affects the position of the graph.

18. State the relationships between the coefficients and the roots of a quadratic equation $ax^2 + bx + c = 0$.

19. Describe how the graph of $f(x) = x^2$ changes if it is transformed to $f(x) = (x - 3)^2 + 2$.

20. List the key features (vertex, axis of symmetry, direction of opening) you would identify when analysing a quadratic function.

Hard Questions

21. For the quadratic equation $2x^2 + (3 - m)x + (m - 1) = 0$, determine the conditions on m such that the equation has two distinct real roots.

22. Prove that for the quadratic equation $ax^2 + bx + c = 0$ with roots r and s, $r + s = -\frac{b}{a}$ and $rs = \frac{c}{a}$.

23. Prove that if a quadratic equation has one real (repeated) solution, then its graph is tangent to the x-axis.

24. Given that a quadratic function has its axis of symmetry at $x = 3$ and passes through the points $(1, 7)$ and $(5, 7)$, determine the vertex of the parabola.

25. Let $f(x) = x^2 + px + q$ be a quadratic function with vertex at $(2, -3)$. Determine the values of p and q.

26. A rectangle has an area given by $A = x(20 - x)$, where x is one side. Determine the value of x that maximises the area and calculate the maximum area.

27. Show that the quadratic function $f(x) = x^2 - 6x + k$ attains a minimum value of -9 when k is equal to a specific value, and find that value.

28. If the two roots of the quadratic equation $ax^2 + bx + c = 0$ differ by 4, show that the coefficients satisfy the relation $\sqrt{b^2 - 4ac} = 4|a|$.

29. Consider the quadratic function $f(x) = ax^2 + bx + c$ where a, b and c are non-zero and $f(1) = 0$. Prove that 1 is one of its roots.

30. For what values of m does the quadratic equation $x^2 + (m - 3)x + m = 0$ have exactly one real solution, and what is that solution?