ILLAWARRA TUTORING

In this worksheet you will explore quadratic equations. You will learn how to manipulate equations of the form $ax^2 + bx + c = 0$ and solve simple cases by isolating the squared term (by taking square roots) and by analysing the discriminant. You will also investigate the relationship between the coefficients and the graph, including key features such as the vertex and axis of symmetry.

Easy Questions

1. Explain what is meant by a quadratic equation. Answer using the standard form $ax^2 + bx + c = 0$ and state any conditions on the coefficients.
2. Rewrite the equation $3 + 2x^2 - 5x = 0$ in the standard form.
3. Solve the equation $x^2 = 25$ by isolating x^2 and taking the square root.
4. Solve the equation $2x^2 = 18$ for x.
5. For the quadratic function $y = x^2 - 4x + 3$, identify the axis of symmetry and state the method you would use to find the vertex.

Intermediate Questions

6. Solve the equation $x^2 - 9 = 0$ by isolating x^2 and applying the square root method.
7. Solve $3x^2 = 27$ for x.
8. Solve the equation $4x^2 - 16 = 0$ by isolating the squared term.
9. Solve the equation $\dfrac{x^2}{9} = 4$ by rewriting it into the form $x^2 = \text{constant}$.
10. For the quadratic equation $x^2 - 4x + 4 = 0$, calculate the discriminant and state what this reveals about the roots.
11. Solve $x^2 = 64$ by the square root method.
12. Solve the equation $2x^2 - 50 = 0$ for x.
13. Solve $x^2 - 2 = 0$ by isolating the square term.
14. Solve the equation $5x^2 = 45$.
15. Explain why the equation $x^2 + 1 = 0$ does not have any real solutions.

16. Determine the axis of symmetry for the quadratic function $y = 2x^2 + 8$.

17. Determine the vertex of the function $y = x^2 - 6$ by explaining how the axis of symmetry helps in locating the vertex.

18. For the function $y = -x^2 + 2$, find its maximum value and indicate where it is attained.

19. For the quadratic function $y = x^2 - 2x$, find the x-intercepts by solving $y = 0$.

20. The equation $x^2 - 6x + k = 0$ has a repeated (double) solution. Determine the value of k and explain your reasoning using the discriminant.

Hard Questions

21. Prove that any quadratic equation of the form $x^2 = c$ can be solved by taking square roots. In your explanation, include a discussion of any restrictions on the constant c.

22. Compare the solutions of the equations $x^2 - 4x = 0$ and $x^2 = 0$. Explain how the inclusion of the linear term affects the solutions.

23. Explain why the quadratic equation $x^2 = 9$ has exactly two solutions when $9 > 0$, and discuss what happens when $c = 0$ in an equation of the form $x^2 = c$.

24. Consider the quadratic equation $x^2 - 4x + m = 0$. Determine the range of values for m for which the equation has two distinct real solutions. Justify your answer.

25. The quadratic equation $x^2 - 2x + 1 = 0$ has a repeated solution. Explain why, in terms of the graph of its function, the parabola touches the x-axis at only one point.

26. Show that for any quadratic equation of the form $x^2 + c = 0$, if $c > 0$ then there are no real solutions. Include a brief discussion based on the properties of square numbers.

27. Consider the quadratic equation $6x^2 + bx + 2 = 0$. Determine the conditions on b such that the discriminant is non-negative and the equation has real solutions.

28. Explain, using algebraic reasoning, why the quadratic equation $(x - d)^2 = 0$ has a single unique solution for any real number d.

29. A general quadratic function is given by $y = ax^2 + bx + c$ and its vertex is (h, k). Derive a formula for h in terms of a and b, and explain the significance of this value.

30. A parabola is defined by the equation $y = x^2 + 2px + p$. Determine the value of p for which the parabola touches the x-axis. Provide a justification for your answer using the discriminant.