ILLAWARRA TUTORING

In this worksheet you will expand your skills to solve systems that include at least one non-linear equation. Work through each problem step by step and show your full working.

Easy Questions

1. Solve the system
$$x + y = 5, \quad xy = 6.$$
Write your answers as ordered pairs.

2. Solve the system
$$y = x^2, \quad x + y = 6.$$
Show all steps leading to the solution.

3. Solve the system
$$x - y = 1, \quad xy = 4.$$
Find the values of x and y.

4. Solve the system
$$x + y = 3, \quad x^2 + y^2 = 5.$$
Hint: Recall the identity $(x + y)^2 = x^2 + 2xy + y^2$.

5. Solve the system
$$y = x^2, \quad y = 2x + 3.$$
Determine the intersection points of the parabola and the line.

Intermediate Questions

6. Solve the system
$$x^2 + xy = 12, \quad y = x + 2.$$
Replace y and solve for x, then find y.

7. Solve the system
$$y = x^2 - 4, \quad x + y = 0.$$
Substitute and solve the resulting quadratic.

8. Solve the system
$$x^2 + y^2 = 25, \quad y = x + 1.$$
Substitute the line equation into the circle equation and find all solutions.

9. Solve the system
$$xy = 8, \quad x + y = 6.$$
Determine the values of x and y by forming a quadratic.

10. Solve the system
$$x^2 - y^2 = 9, \quad x - y = 3.$$
Use factorisation to reduce and solve the system.

11. Solve the system
$$y = x^2 + 2x, \quad xy = -8.$$
Substitute the expression for y into the second equation and solve for x.

12. Solve the system
$$x^2 + y = 7, \quad y^2 + x = 11.$$
Use substitution techniques to find all possible solutions.

13. Solve the system
$$xy = 5, \quad x^2 + y^2 = 29.$$
Hint: Use $(x + y)^2 = x^2 + 2xy + y^2$ to help solve.

14. Solve the system
$$x^2y = 16, \quad x + y = 5.$$
Express y in terms of x and solve the resulting equation.

15. Solve the system
$$y = x^2 - 2x - 3, \quad xy = -4.$$
Substitute y into the product equation and solve for x.

16. Solve the system
$$x^2 + 4y^2 = 4, \quad x = 2y.$$
Substitute x into the ellipse equation and solve for y.

17. Solve the system
$$y^2 = x + 4, \quad x = y + 2.$$
Substitute the expression for x into the quadratic equation and solve.

18. Solve the system
$$xy + x = 6, \quad y = x + 1.$$
Factor where possible and determine the solutions for x and y.

19. Solve the system
$$\frac{1}{x} + \frac{1}{y} = 1, \quad x - y = 1.$$
Express the first equation as $\dfrac{x+y}{xy} = 1$ and solve the system.

20. Solve the system
$$x^2 + xy + y^2 = 19, \quad x + y = 5.$$
Express $x^2 + xy + y^2$ in terms of $(x + y)$ and xy to find the solutions.

Hard Questions

21. Solve the system
$$y = x^2 - 3x + 2, \quad y^2 = 4x.$$
Substitute the expression for y into the second equation and solve the resulting equation.

22. Solve the system
$$x^2 + y^2 = 10, \quad x^2 - y = 2.$$
Express y from the second equation and substitute into the first.

23. Solve the system
$$y = x^2, \quad y^2 = 4x + 8.$$
Substitute and solve for x, then determine y.

24. Solve the system
$$x^2y + y^2 = 20, \quad x + y = 4.$$
Express y in terms of x from the linear equation and substitute into the non-linear equation.

25. Solve the system
$$\sqrt{x+1} + y = 5, \quad y^2 = x + 9.$$
Isolate the square root and square appropriately, then solve for x and y. (Remember to check for extraneous solutions.)

26. Solve the system
$$x^2 + y^2 = 2xy + 1, \quad x - y = 1.$$
Rewrite the first equation in a recognisable perfect-square form and solve.

27. Solve the system
$$(x + y)^2 = 16, \quad xy = 3.$$
Consider both possibilities for $x + y$ and solve the resulting quadratic.

28. Solve the system
$$x^3 + y^2 = 10, \quad x + y = 3.$$
Express one variable in terms of the other and use substitution to solve the higher degree equation.

29. Solve the system
$$y = x^2 - x, \quad x^2 + y^2 = 10.$$
Substitute the expression for y into the circle equation and solve for x.

30. Solve the system
$$x^2y - 2xy + y = 0, \quad x + y = 4.$$
Factor the first equation as $y(x-1)^2 = 0$ then use the linear equation to determine the solutions.